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A Biased Inferential Naivety Model for Agents' Learning in Social Networks

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Abstract—**We proposed a model of learning and belief formation in which a group of agents tries to learn the true underlying state of the world and make the best possible decisions. Agents with limited computational ability, in addition to receiving noisy private signals, observe the decisions of their neighbors. It is well known that Bayesian inference is very complex in social observations, especially when agents are unaware of the structure of the social network. In our model, the role of knowledge derived from the social observations of each agent is separated from that's of her private observations in the formation of her belief. Thus, to reduce the complexity of Bayesian inference, the processing of social observations is approximated using the inferential naivety assumption. With this assumption, agents naively believe that each neighbor's decisions are based solely on his or her private observations and that their social interactions are ignored. Another important initiative in the proposed model is to eliminate herd behavior by introducing an exponential bias and reducing the weight of early social observations compared to recent observations. A number of Monte Carlo simulation experiments confirm the features of the proposed model. This includes asymptotic learning of all agents and increased learning efficiency in social networks.**

Keywords: Bayesian decision making; Heuristic method; Inferential naivety; Rational agents; Social learning model.

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I. INTRODUCTION

It is clear that humans are naturally social and influence each other's beliefs, choices and behaviors. The process of obtaining information and updating beliefs in the context of social networks is called social learning. Nowadays, with the expansion of online social media, people can quickly and easily exchange large amounts of different types of content such as opinions, choices and behavior of others anywhere in the world. Therefore, engineers, economists, and social scientists are interested in studying learning on social media and determining how beliefs and behaviors evolve over time. Engineers study the social learning in the context of distributed signal processing in the wireless sensor network for distributed detection or to solve

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coordination and consensus problem in control theory [1]-[5]. Economists, politicians, and sociologists classically try to identify the factors that influence people's beliefs to predict and control individual and public behavior [6]-[11]. Issues related to the spread of misinformation are also another field of study. In society, there may be some sources of misinformation either by parties that deliberately try to manipulate beliefs or because some agents and leaders of society are stubborn and they will not change their minds. So, it becomes important to understand what kind of societies and social structures are "strong" against the spreading misinformation, and what can be done to strengthen society and make ideas more stable so that they are less exposed to manipulation [12]-[15].

In this paper, we present a biased inferential naivety social learning model (BIN). This paper addresses the question of how to reduce the complexity of a Bayesian inference on networks and still ensure learning for all the agents. It is assumed that a set of n agents interact in the context of a strongly connected graph. At each time step t, each agent receives a private signal from the environment and observes the neighbors' decisions at time $t-1$. The set of private signals received from each agent is called private information and the set of information received from neighbors is called agent social information. Each agent has a belief about the true state of the world that evolves during the time with receiving new information. The decision of each agents is made according to her belief in order to maximize the utility function. Beliefs are represented by a probability distribution on the set of possible states.

Our first innovation is to reduce the computational complexity of Bayesian inference in social observations. In the proposed model, the role of knowledge obtained from each agent's social observations is separated from her private observations. In this way, whereas the fully Bayesian inference is used for private observations, the social observations are approximately inferred using the inferential naivety assumption [16]-[17]. According to this assumption, agents do not reason about other agents' social observations, and neighbors' decisions are considered just as a result of observing private signals. The information obtained from the processing of social observations combines with private signals to form the agent's belief. Our second innovation is to reduce herd behavior by underweighting initial social observations using exponential bias. Herd behavior occurs when an agent does not consider its private observations in making decisions. When all agents do the herd behavior the information cascade occurs and learning stops [3]. Our experiments show that using the BIN model, herd behavior reduces, learning continues until all agents (even the uninformed ones) learn the truth and the learning performance improves.

The paper is organized as follows. In section 2, the social learning models and their characteristics are described. In section 3, the main categories of social learning models are given. In section 4, the notation using in our model is introduced. In section 5, the Biased Inferential Naivety model is explained. Finally, in section 6, a number of Monte Carlo simulation experiments confirm the features of the proposed model. Experiments show that our model outperforms the proposed model without bias, the BWR model, and the learning model based only on private observations.

II. SOCIAL LEARNING MODELS

In this section, we briefly express the characteristics of the social learning models [11]. In social learning models, each agent has an Initial view or prior beliefs that are the likelihood of each possible state of the world. In many studies, it is assumed that all agents have the same and non-informative priors, and in some papers, the use of empirical probabilities is suggested. The next characteristic of social learning model is the signal structure. The signal structure determines the amount of information the signals generate to identify the underlying state for each agent. Usually in social networks, agents have different observational abilities and some of them get more informative signals1 and are called strong agents. These agents can help other agents learn faster. On the other hand, for some agents, some states may be observationally equivalent to the true state. These agents are not able to distinguish the correct state in isolation and need gain information from society. The information can be obtained from observing the other decisions or through communicating with others about their beliefs. The structure of the social network affects the amount of information received by agents through the community and is specified in each model of social learning. This can be definite or accidental. The method of information processing is the most important characteristic of a social learning models. It determines how each agent incorporates current information with the newly received information. In the following section, some of the information processing methods are briefly introduced.

III. BAYESIAN AND HEURISTIC MODELS

Accordingly, social learning models can be divided into Bayesian [3], [7], [10], [18], [19] and heuristic [20]-[28] categories. In Bayesian models, agents must repeatedly apply the Bayesian rule on their private and social observations over time. They must infer about the global signal structure. This inference will be complicated and hard to analysis in complex environments with a large number of agents. In the case that agents are unaware of the structure of the global network, this method will be unusable due to its complexity. To avoid the complexity of the Bayesian models several heuristic models are proposed in the literature. Decision rules of heuristics are usually simple functions. Many of them are imitation based in which agents randomly follow the behavior of an observed agents or accept a combination of their observations. For example, in the famous Degroot's model [29], each agent's belief is updated as a convex

[1] The expected informativeness of the agent's private signals is shown by the relative entropy of the true state relative to the false state according to the signal structure of that agent. The more informative is the agent's signals, the higher is his ability to observe.

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combination of her neighbors' beliefs. These heuristic models have weak theoretical foundations and their predictions mostly depend on their specific learning rules rather than behavioral assumptions. Another group of heuristic models, is called as Bayesian heuristic [30]-[32] which is between the Bayesian models and the imitation based models. In these models, the fully Bayesian inference is simplified by using some heuristic assumptions. Therefore, the complexity of the fully Bayesian inference is reduced, but some features of Bayesian learning are still preserved. It seems that people's decision-making is a kind of Bayesian heuristic. People tend to behave optimally, but because of cognitive characteristics, they are satisfied with choices that are good enough instead of optimal.

The Bayesian without recall model (BWR) [25], [30]-[32] is a heuristic Bayesian model in which it is assumed that for all future time-steps, agents replicate the first step of the full Bayesian inference to their recent observations from a common prior. We compare the BWR model with the BIN model in the simulation section.

The locally Bayesian learning model is another heuristic Bayesian model in which each agent extracts new information using the full history of observed reports in her local network [33]. In this model, it is assumed that from the point of view of each agent, her local network is the whole network. This model learns the truth just in the network that is a social quilt. Also, the agents need extra memory to preserve all actions of neighbors from the first time. Whereas in our model agents learn the truth in all connected networks and no need for extra memory.

IV. The Notation

In our model, there are n agents A_i, interacting in the context of directed graph $G = (N, E)$, refered as social network, where N corresponds to the agents indexed in $[n] = \{1, 2, \cdots, n\}$ and $E \subseteq [n] \times [n]$ is the set of all directed edges. Each edge shows a directed interaction between two corresponding agents. The neighbors of A_i, denoted by $\mathcal{N}(i)$, is the set of all agents A_j such that there exists an edge from A_j to A_i. It is assumed that the agents do not know the global structure of the graph G, but each agent A_i knows her own neighbors. Let the binary space $\Theta = \{0,1\}$ be the set of possible states of the world and the unknown true state $\theta \epsilon \Theta$ chosen by the nature and assume that both values of θ are equaly likely. Suppose that the time proceeds in discrete steps and is indexed by $t \in \{0,1, \ldots\} = \mathbb{N}_0$. For each agent A_i, the set S_i denotes her finite signal space, and conditional on the state of the world θ, the private signal sequence $(s_i^t)^{t\in\mathbb{N}_0}$ of the agent A_i is generated identically and independently according to a probability mass function $\ell_i(.|\theta)$ on S_i during the time. The probability mass function $\ell_i(.|\theta)$ depends on the parameter θ and is refered as the signal structure or likelihood function of A_i. We assume that the private signals s_i^t are also independent across the agents and no single signal indicates the underlying state. In each time period $t \in \mathbb{N}_0$, each agent A_i, in addition to private observation s_i^t, receives the choices of her naighbors at the previouse time $t-1$. Let $\alpha_i^t \in X, t \in \mathbb{N}$ denote the action choosen by A_i in time t, where $X = \{0,1\}$ is the same action space of all agents $A_i, i \in [n]$.

V. A Biased Inferential Naivety Model

As mentioned before, an agent's belief about the true state of the world is represented by a probability distribution on Θ. Let β_i^t be the A_i's belief in true state $\theta = 1$ at time t that is refined based on all the information that has been available to her by the time t. After forming a belief, A_i chooses her action according to a decision rule maximizing her expected payoff. The decision rule deterministically or randomly maps the agent's belief to the action space. According to our deterministic decision rule, at the time t, if $\beta_i^t \geq 1/2$ then A_i choose the decision $\alpha_i^t = 1$, and otherwise choose $\alpha_i^t = 0$. Now assume that $I_i^t := \{s_i^{0:t}, \alpha_{\mathcal{N}(i)}^{0:t-1}\}$ is the set of all information available to A_i up to time step t, where $s_i^{0:t}$ is the history of private signals observed by A_i in the time steps $o, 1, \ldots, t$ and $\alpha_{\mathcal{N}(i)}^{0:t-1}$ denotes the decisions made by A_i 's neighbors in the time steps $0, 1, \ldots, t-1$. We respectively call them as the private and social observations of A_i up to time t. Using the Bayesian rule and the fact that $(s_i^t)^{t\in\mathbb{N}_0}$ are generated independently and are also independent of $\alpha_{\mathcal{N}(i)}^{0:t-1}$, β_i^t is formulated as

$$\begin{aligned}
\beta_i^t &= p(\theta = 1|I_i^t) = p\left(\theta = 1|\alpha_{\varkappa(i)}^{0:t-1}, s_i^{0:t}\right) \\
&= \frac{p\left(\theta = 1, s_i^{0:t}|\alpha_{\varkappa(i)}^{0:t-1}\right)}{p\left(s_i^{0:t}|\alpha_{\varkappa(i)}^{0:t-1}\right)} \\
&= \frac{p\left(s_i^{0:t}|\alpha_{\varkappa(i)}^{0:t-1}, \theta = 1\right)p\left(\theta = 1|\alpha_{\varkappa(i)}^{0:t-1}\right)}{\sum_{k\in\{0,1\}} p\left(s_i^{0:t}|\alpha_{\varkappa(i)}^{0:t-1}, \theta = k\right)p\left(\theta = k|\alpha_{\varkappa(i)}^{0:t-1}\right)} \\
&= \frac{p\left(s_i^{0:t}|\theta = 1\right)\pi_i^t}{p\left(s_i^{0:t}|\theta = 1\right)\pi_i^t + p\left(s_i^{0:t}|\theta = 0\right)(1-\pi_i^t)} \\
&= \frac{1}{1 + \mathcal{R}_i^t \frac{1-\pi_i^t}{\pi_i^t}}
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
\mathcal{R}_i^t &= \frac{\ell_i(s_i^{0:t}|\theta = 0)}{\ell_i(s_i^{0:t}|\theta = 1)} \\
&= \begin{cases} \mathcal{R}_i^{t-1} \times \frac{\ell_i(s_i^t|\theta = 0)}{\ell_i(s_i^t|\theta = 1)}, & t \geq 1 \\ \frac{\ell_i(s_i^0|\theta = 0)}{\ell_i(s_i^0|\theta = 1)}, & t = 0 \end{cases}
\end{aligned} \tag{2}$$

is the product of likelihood ratio of the private signals $s_i^{0:t}$, and $\pi_i^t = p\left(\theta = 1|\alpha_{\mathcal{N}(i)}^{0:t-1}\right)$, refered as A_i's social belief, represents the Bayesian belief on $\theta = 1$ based on all of her social observations up to time t. In other words, the result of all social interactions has been summarized in the content of social belief π_i^t, and according to (1), it is considered as a prior distribution to determine the posterior belief β_i^t. In the proposed model, as the role of social and private observations in

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the belief formation are separated, it would be possible to tune their impact independently. Thus, unlike the Bayesian method, agents can behave differently under the same conditions. In the real world, individuals are also different in decision making. For example, an individual can be a leader and have more trust in her own observations or, can be a follower and give more credit to others. Now using Bayesian rule the social belief of A_i at time t is updated as

$$\pi_i^t = \frac{\sigma_{i,1}^{t-1}\pi_i^{t-1}}{\sigma_{i,1}^{t-1}\pi_i^{t-1} + \sigma_{i,0}^{t-1}(1-\pi_i^{t-1})}, \quad (3)$$

where $\sigma_{i,k}^t = p\left(\alpha_{\mathcal{N}(i)}^t \middle| \theta = k,\ \alpha_{\mathcal{N}(i)}^{0:t-1}\right), k \in \{0,1\}$ denotes the probability that A_i's neighbors make decisions $\alpha_{\mathcal{N}(i)}^t$ conditional on $\alpha_{\mathcal{N}(i)}^{0:t-1}$ and on the true state $\theta = k \in \{0,1\}$. To compute $\sigma_{i,k}^t$, A_i should estimate the possible information set $\mathcal{I}_{i,j}^t$ which is the set of all signals contained in S_j that could lead to the decision α_j^t from A_i's point of view, for all $A_j, j \in \mathcal{N}(i)$. The initial social belief π_i^0 represents A_i's bias on $\theta = 1$, but we assume that $\pi_i^0 = {}^1/_2$ for all $i\epsilon[n]$, which means that all prior beliefs are interior [34]. Each agent has to extract possible information sets that lead to the decisions $\alpha_{\mathcal{N}(i)}^t$. At the first step, each agent A_i observes her signal s_i^0 that is generated with her likelihood function $\ell_i(.|\theta)$ and forms her initial belief β_i^0 according to the Bayesian formula

$$\beta_i^0 = \frac{\pi_i^0 \ell_i(s_i^0|\theta=1)}{\pi_i^0 \ell_i(s_i^0|\theta=1) + (1-\pi_i^0)\ell_i(s_i^0|\theta=0)}. \quad (4)$$

Then, each A_i makes her first decision x_i^0. At any time step $t \geq 1$ each agent receives decisions made by her neighbors at the previous time and observes her private signal. Subsequently, at each time step t, each agent A_i's information set is extended with realized signal s_i^t and also with $\left(\alpha_j^{t-1}\right)_{j\in\mathcal{N}(i)}$.

At the second step, the information set of A_i is $I_i^1 = \left\{s_i^0, s_i^1, \alpha_{\mathcal{N}(i)}^0\right\}$. Each A_i calculates her social belief π_i^1 based on her first social observations $\alpha_{\mathcal{N}(i)}^0$ using (3). As the first decisions of agents are taken only on the basis of their own private signals, the probability of each neighbor's decision is independent on others decisions and we have

$$\sigma_{i,k}^0 = \prod_{j\in\mathcal{N}(i)} p\left(\alpha_j^0|\theta=k\right), k \in \{0,1\}. \quad (5)$$

Then, in order to estimate the possible information set $\mathcal{I}_{i,j}^0$, it is enough to divide each neighbor's signal space into two parts S_j^1 and S_j^0, where S_j^1 (resp. S_j^0) contains the signals observed by A_j which led to making the decision $x_j^0 = 1$ (resp. $x_j^0 = 0$). Hence, we have

$$\mathcal{I}_{i,j}^0 = \begin{cases} S_j^1 & \alpha_j^0 = 1, \\ S_j^0 & \alpha_j^0 = 0. \end{cases} \quad (6)$$

Considering (4) and our decision rule, the sets S_j^1 and S_j^0 [30] will be as

$$S_i^1 = \{s \in S_i : \ell_i(s|\theta=1)\pi_i^0 \geq \ell_i(s|\theta=0)(1-\pi_i^0)\}$$

And $S_i^0 = S_i \backslash S_i^1$. (7)

Hence for each $j \in \mathcal{N}(i)$ we have

$$\sigma_{i,k}^1 = \prod_{j\in\mathcal{N}(i)} \left(\sum_{s_j\in S_j^1} \ell\left(s_j|\theta=k\right)\right)^{\alpha_j^0\left(\alpha_j^0+1\right)/2} \times \left(\sum_{s_j\in S_j^0} \ell\left(s_j|\theta=k\right)\right)^{1-\alpha_j^0}, \quad (8)$$

where $k \in \{0,1\}$.

By computing π_i^1 from (3) and $\mathcal{R}_i^1$ from (2), the A_i's belief β_i^1 is determined using (1) and the second action α_i^1 is taken.

At the third step, A_i's information set is extended by $\left\{s_i^2, \alpha_{\mathcal{N}(i)}^1\right\}$. Since for each $j \in \mathcal{N}(i)$, α_j^1 depends on both A_j's private signals and A_j's neighbors' decisions at first step, to compute $\sigma_{i,k}^1$, each agent A_i, in addition to estimating the possible information set of her neighbors should also estimate the possible information set of the neighbors of her neighbors. It is clear that as time goes by, this inference should be made for the neighbors at the longer paths on the social network. Since agents observe only their own immediate neighbors' decisions and are not aware of the general structure of the social network, fully Bayesian inference will be very complicated.

The inferential naivety assumption [16] is used for inferring the social observation to reduce the complexity of the Bayesian inference. According to the inferential naivety assumption, agents naively believe that the decisions of neighbors in all times are just due to their private observations and they ignore that their neighbors also have social interactions that may influence their decisions (Fig. 1).

Therefore, for all $t \geq 1$, we have

$$\sigma_{i,k}^t = \prod_{j\in\mathcal{N}(i)} \left(\sum_{s_j\in S_j^1} \ell\left(s_j|\theta=k\right)\right)^{\alpha_j^t\left(\alpha_j^t+1\right)/2} \times \left(\sum_{s_j\in S_j^0} \ell\left(s_j|\theta=k\right)\right)^{\left(1-\alpha_j^t\right)}. \quad (9)$$

where $k \in \{0,1\}$.

According to [17] using the inferential naivety assumption, the probability of herd behavior[2] in dense graphs increases. It is because the agents do not consider the repetition of the information and place too much weight on early signals. If these early signals are inaccurate, the learning may never happen at all. To eliminate this kind of herd behavior, we use the biased Bayesian inference [35] with an exponential bias in determining the social beliefs, so the formula (3) becomes as

$$\pi_i^t = \frac{\sigma_{i,1}^{t-1}(\pi_i^{t-1})^{B_f}}{\sigma_{i,1}^{t-1}(\pi_i^{t-1})^{B_f} + \sigma_{i,0}^{t-1}(1-\pi_i^{t-1})^{B_f}}, \quad (10)$$

where B_f is the exponential bias on the prior probability that is called forgotten bias. If $0 < B_f < 1$,

[2] An agent A_i makes herd behavior at time t if she makes decision without considering her private signal, i.e., $p\left(\alpha_i^t = 1 \middle|\ \alpha_{\mathcal{N}(i)}^{0:t-1}, s_i^{0:t}\right) = p\left(\alpha_i^t = 1 \middle|\ \alpha_{\mathcal{N}(i)}^{0:t-1}, s_i^{0:t-1}\right)$.

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then the prior distribution will be flatter than the original one and whatever social observations get older, it will have less impact on the π_i^t. So, the influence of early signals decreases over time and as a result, the herd behavior is also reduced. If the forgotten bias is determined for each agent separately, each agent could have its own behavior and two agents with the same information setting may make different decisions.

In the Bayesian without recall model also, the first step of Bayesian inference is repeated for further steps but using the BWR each agent at each step uses the most recent observations and ignores previous observations. These observations that are neglected in the BWR model (especially the history of private signals) are informative and the performance of model improve by using all observations. In our model, all history of social observations is used and all private observations are processed without any approximation. We compare the BWR model with our model in simulation section. The result show that the BWR method can lead to a lack of learning or even a failure to reach consensus in many situations even in rich information settings but the BIN model led to learning. It is noteworthy that since in our model, social beliefs and likelihood ratios are recursively updated as in equations (2) and (3), there is no need for extra memory to keep track of observations and the amount of memory remains constant over time.

VI. SIMULATION

In this section, we illustrate some features of the proposed model, by Monte Carlo simulations. The performance of the proposed model (BIN) is compared with the following three models:

- Individual Learning (IL): In this model, agents do not have any social interaction and just learn through their private observations. Comparison of BIN with IL confirms that learning occurs much faster using social observations and the proposed model.

- Inferential Naivety (IN): In this model, agents' beliefs are updated using equations (1), (2), (3) and (7) (the forgetting bias is considered $B_f = 1$). Comparison of BIN with IN confirms that in our model, using forgotten bias reduces the herd behavior.

- Bayesian without Recall (BWR): This model is implemented according to [30, Sec. III]. Experiments show that with the BWR model, the probability of correct consensus is very low and even in many cases consensus does not occur. But in our model, the correct consensus is reached in all cases.

In our simulations, the number of agents is $n = 10$, each agent at each time slot receives a random signal according to her likelihood function $\ell_i(.|\theta_1)$. Our intention of belief is the belief that the true state of the world is $\theta = 1$. The expected value of agents' beliefs in each time slot is calculated by Monte Carlo method as the average of 2000 trials and each trial takes 6000 time slots. In BIN model, in all experiments we set $B_f = 0.95$. We test efficiency of mentioned models in four different combinations of the agents' observational ability and the social networks' structure. In the first experiment, all agents are weak except two of them. Agents interact in directed connected graphs. In the second experiment, the same agents as in the first experiment interact in dense connected graphs. In the third experiment, all the agents are weak and interate in dense connected graphs. In the fourth experiment, all agents are strong and interact in directed connected graphs.

The results of experiments are represented by six types of figures, which are explained below.

-The observational ability of each agent: These figures show the evolution of the expected belief of each agent using IL method, up to time $t = 400$. Because, using the IL method, agents' beliefs evolve only on the basis of private signals, the expectation used with IL is a good option for displaying observational abilities (In Fig. 2, A_2 has low observational ability and A_1 has high observational ability)

-The evolution of society's belief: These figures represent, the average of all agents' expected beliefs over time, using IL, IN and BIN models. So, the growth of society's expected belief can be compared in three models. For example, in all experiments, the BIN curve grows faster than the IL curve that indicates social learning using the BIN is more efficient than individual learning using IL.

-The evolution of each agent's expected belief using IL, IN and BIN models: In these figures, the evolution of each agent's expected belief using three models is shown separately in ten sub plots. Each plot shows the impact of learning models on the evolution of each agent's belief. For example, in Fig.4, using the BIN model, interaction with weak agents does not negatively affect the ability of strong agents A1, A10 .

-The probabilities of consensus on the correct decision, consensus on the wrong decision and lack of consensus using IL, IN and BIN models: These bar diagrams represent the ratio of the number of reaching consensus on correct decision (respectively for consensus on the incorrect decision and failing to reach a consensus) to the total number of 2000. In many applications, such as distributed detection systems, which aim to gather scattered information between agents, lack of consensus is better than the wrong consensus.

-The histograms of decisions in time slots 300, 3000, 6000: These figures represent the histograms of decisions of all agents in the time slots $t = 300, t = 3000$ and $t = 6000$ using IN and BIN. These figures confirm the continuous increasing the number of correct decisions in society using the BIN method and also the occurrence the herd behavior using the IN model.

Now, in the following, four experiments are described in more details.

- First experiment: This experiment confirms that using the BIN model, there is no herd behavior and that all agents learn the truth. In this experiment the agents with observational abilities as in Fig. 2, interact in a connected graph. All agents except A_1 and A_{10} have weak observational abilities. Both agents are moderate but the agent A_{10} is stronger than A_1.

In Fig. 3 it is clear that the expected value of society's belief using IN increases for a while then

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remains stable. This indicates that herding occurs for all agents and learning stops. On the other hand, using the proposed model, the expected belief increases steadily and its curve is higher than the expected belief using IL.

In Figure 4, we see that for each agent, the belief curve using BIN is higher than the belief curves using IN and IL, and also its slope is higher. It is also clear that using the BIN model, the weak agents' beliefs with higher social interactions have a higher growth rate. For example, both A_5 and A_6 are weak, but since the information that agent A_5 receives from the society is more than that of agent A_6, the growth of her BIN belief is much higher than her IL belief.

In Fig. 5 using the BIN method, the probability of consensus on the wrong decision is zero because the herd behavior does not happen. The probability of failing to reach a consensus is due to the wrong decisions made by weak agents who have not had enough time to gather information. Using IN, the consensus is almost reached, but more than half of them are the wrong consensus. According to [16] and [17], some of the agents receive incorrect initial signals and due to the occurrence of the herd behavior, ultimately all agents choose the wrong decision.

The second row in Fig. 6 shows that in the BIN, the number of correct decision increase over time, and learning continues over time until the complete confidence is attained. But the first row shows that in the IN model, the decisions of the agents do not change with receiving the new observations, and learning stops very early.

- Second experiment: The results show that in the BIN mode, learning performance is greatly improved by increasing social interactions. In this experiment, the agents of the first experiment interact in a dense connected graph. Figure 7 shows that in the BIN model, the expected value of society's belief increases to 1 with a steeper slope than the one of the previous experiment. Figure 8 shows that in the BIN model, due to the increase in social interactions, all agents, either weak or strong, reach full belief very soon. By comparing Fig. 9 and Fig. 5, it is obvious that in experiment 2, the number of times in which convergence does not occur has decreased. By comparing Fig. 6 with 10, it is clear that in the second experiment, the number of correct decisions have grown more rapidly over the time.

- Third experiment: This experiment confirms that even when all agents are weak, our model works well and the dispersed information is gathered efficiently. In this experiment, all agents are weak. They interact in a dense connected graph. Figs. 11-15 confirm that using the BIN all weak agents learn true so fast. In Figs. 12 and 13, it can be seen that using BIN, learning does not stop and the average belief of society and of each agent steadily increase to belief 1. Comparing Figs. 12 and 13 with Figs. 7 and 8 show that the higher is the number of weak agents, the longer it takes for the society to a consensus on the correct decision. In Fig. 14, the small probability of the wrong consensus is due to time constraints. Figure 15 also confirms the continuous learning and the lack of herd in the BIN model.

Fourth experiment: This experiment compare the BWR model with the BIN model. In this experiment, all agents have strong observable abilities (Fig. 16) and interact in connected graph. In such a rich environment, It is expected that an efficient learning model perform well and detect the correct situation quickly. Figure 17 shows that the performance of IN, IL, BIN is very good but the performance of BWR model is very poor. Since, in the BWR model, agents ignore their previous observations and just consider the last ones. These observations that are neglected in BWR (especially the history of private signals) are informative and useful. In our model all observations is used recursively. Table. 1 shows the probabilities of consensus on the correct decision, consensus on the wrong decision and the absence of consensus using BWR and BIN in all experiments. It confirm the weakness in the learning and even in the agreement on a same decision.

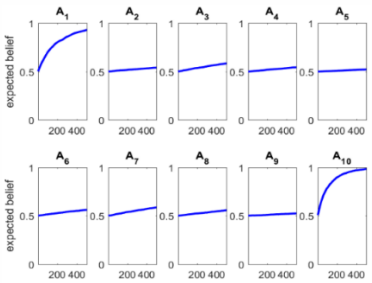

Fig. 2. The observational ability of each agent using the IL (Exp.1).

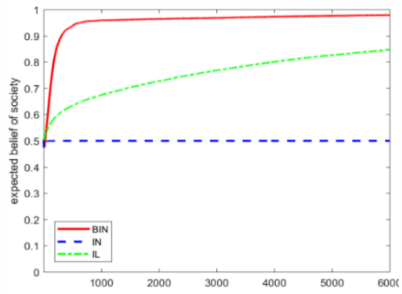

Fig. 3. Evolution of each agent's expected belief using IL, IN and BIN (Exp.1).

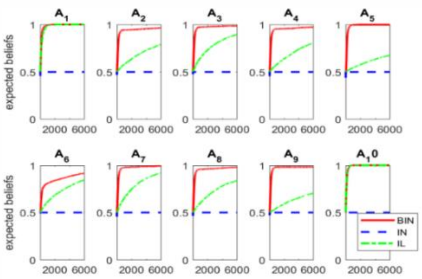

Fig. 4. Evolution of each agent's expected belief using IL, IN and BIN (Exp.1).

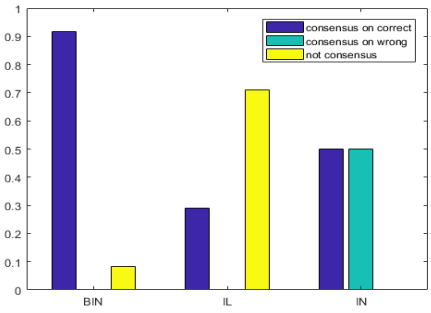

Fig. 5. Probabilities of consensus on the correct decision, consensus on the wrong decision and lack of consensus using IL, IN and BIN (Exp.1).

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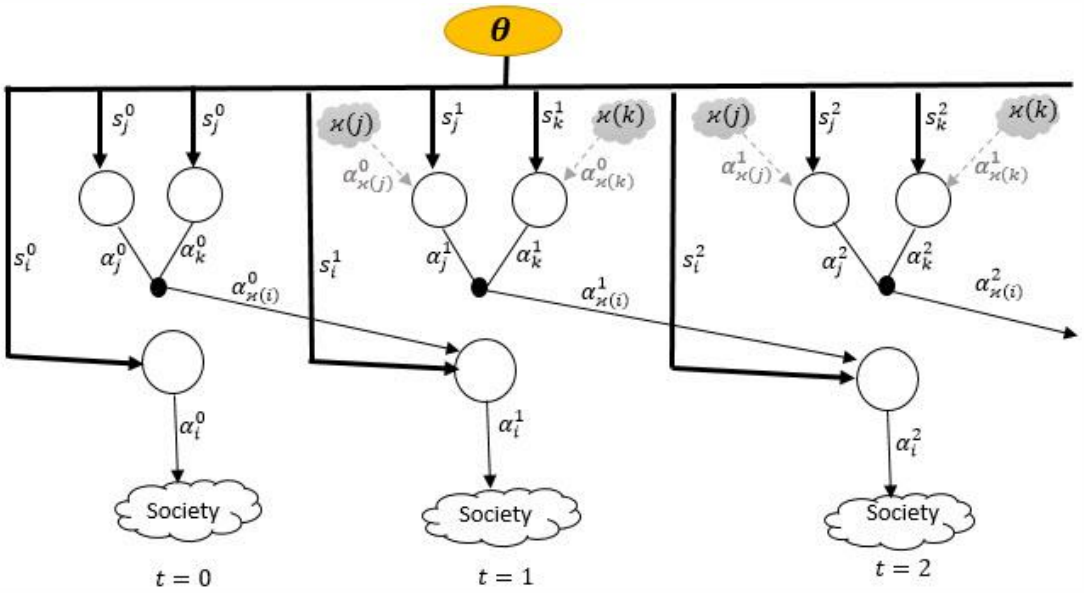

Fig. 1. At each time step, each agent receives a private signal from the true state Θ and observes the neighbors' decisions at time $t-1$. According to the inferential nativity assumption, each agent A_i naively believe that the decisions of her neighbors A_j and A_k are just due to their signals and ignore the their social interaction.

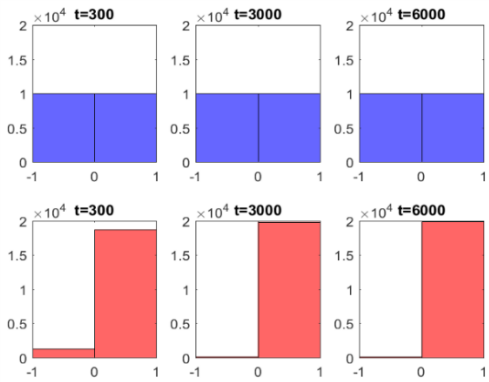

Fig. 6. Histograms of decisions in time slots 300, 3000, 6000 (Exp.1).

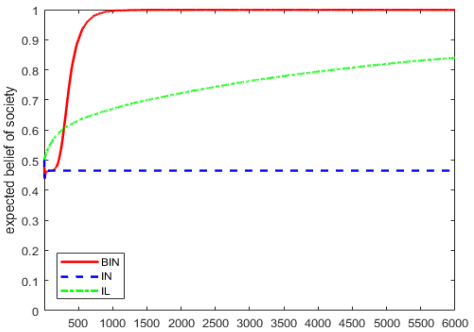

Fig. 7. Evolution of society's belief using IL, IN and BIN (Exp.2).

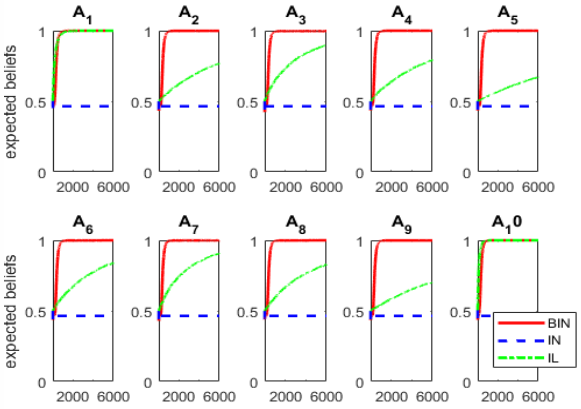

Fig. 8. Evolution of each agent's expected belief using IL, IN and BIN (Exp.2).

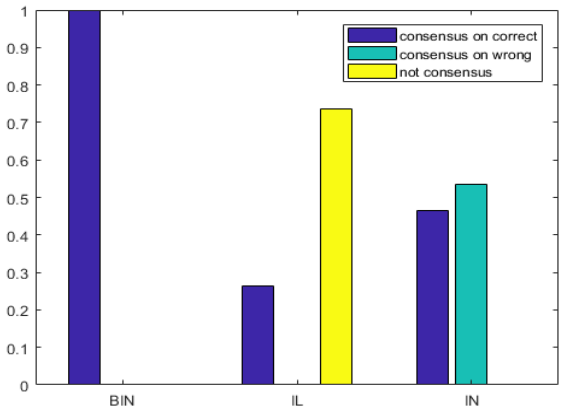

Fig. 9. Probabilities of consensus on the correct decision, consensus on the wrong decision and lack of consensus using IL, IN and BIN (Exp.2).

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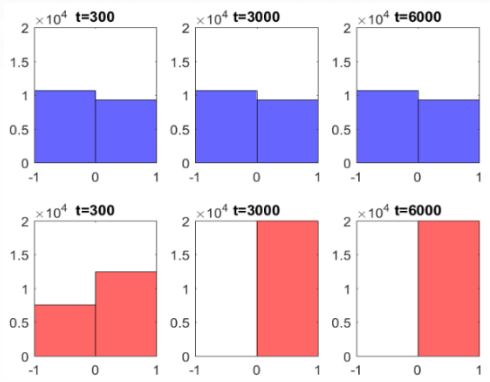

Fig. 10. Histograms of decisions in time slots 300, 3000, 6000 (Exp.2).

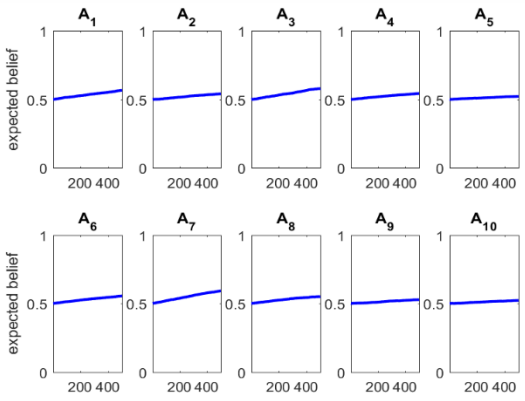

Fig. 11. The observational ability of each agent using the IL (Exp.3).

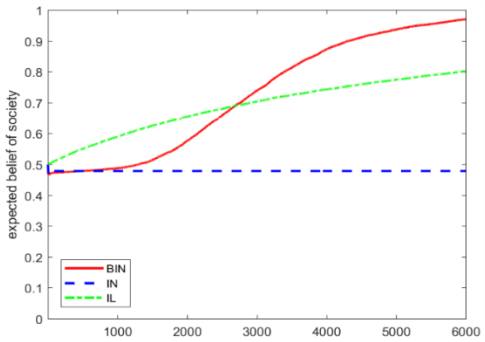

Fig. 12. Evolution of society's belief using IL, IN and BIN (Exp.3).

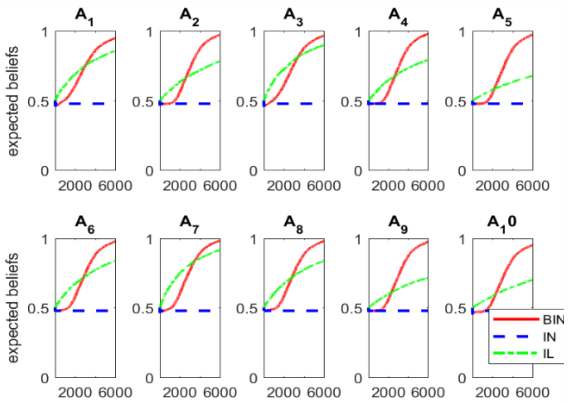

Fig. 13. Evolution of each agent's expected belief using IL, IN and BIN (Exp.3).

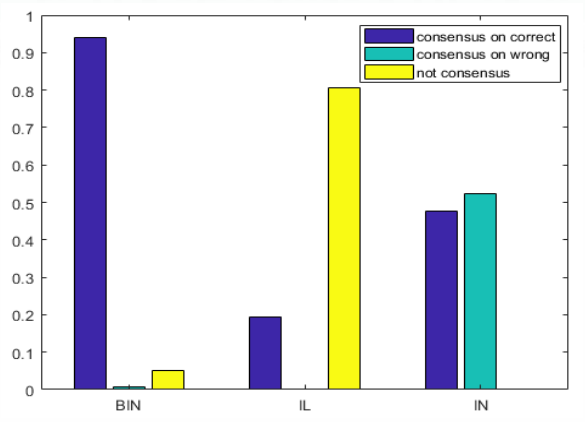

Fig. 14. Probabilities of consensus on the correct decision, consensus on the wrong decision and lack of consensus using IL, IN and BIN models (Exp.3).

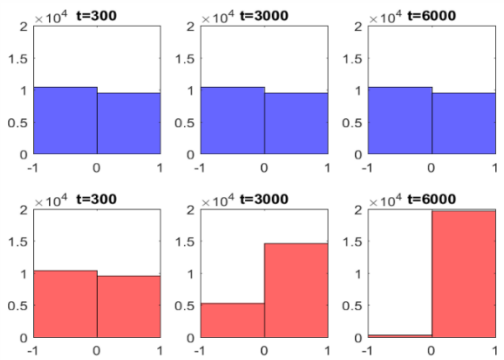

Fig. 15. Histograms of decisions in time slots 300, 3000, 6000 (Exp.3)..

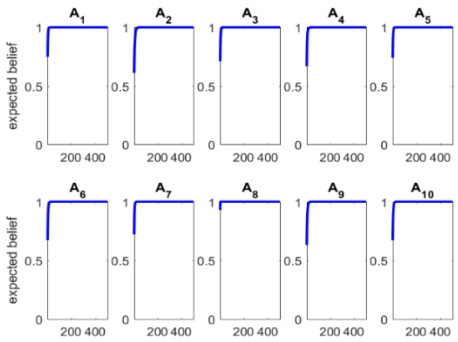

Fig. 16. The observational ability of each agent using the IL (Exp.4).

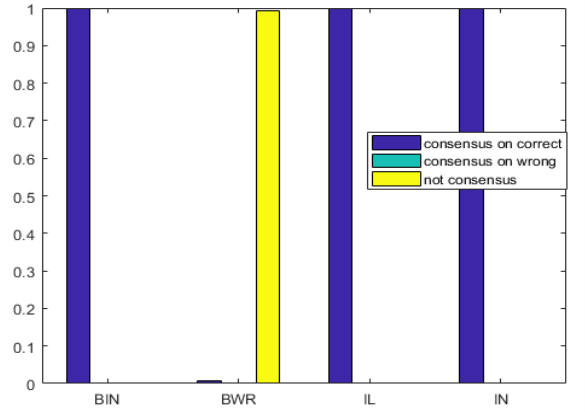

Fig. 17. Probabilities of consensus on the correct decision, consensus on the wrong decision and lack of consensus using IL, IN, BIN and BWR (Exp.4).

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TABLE I. PROBABILITIES OF CONSENSUS ON THE CORRECT DECISION, CONSENSUS ON THE WRONG DECISION AND LACK OF CONSENSUS.

	BWR model			BIN model		
	Cons. on correct	Cons. on wrong	No Cons.	Cons. on correct	Cons. on wrong	No Cons.
Exp_1	0.0	0.0	1	0.92	0.0	0.08
Exp_2	0.0	0.03	0.97	1.0	0.0	0.0
Exp_3	0.44	0.56	0.0	0.94	0.01	0.05
Exp_4	0.01	0.0	0.99	1.0	0.0	0.0

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