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# A Low-Complexity Cyclostationary-Based Detection Method for Cooperative Spectrum Sensing in Cognitive Radio Networks

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*Abstract*— **Fast reliable spectrum sensing (SS) is a crucial problem in the cognitive radio systems. To address this issue, cyclostationarity-based detection methods, which are generally more complex but more reliable than energy detection methods, have been proposed. This paper presents a new method to detect the presence of the second-order cyclostationarity in the OFDM-based primary user (PU) signals. The proposed method has a low computational complexity, while it presents a performance close to the well-known GLRT-based Dandawaté-Giannakis's method. Moreover, since the proposed low-complexity method is robust against the noise uncertainty, it can be a good alternative to the energy detection method. We further propose some cooperative spectrum sensing methods to improve the detection performance. Extensive simulation results confirm the superiority of the proposed schemes.**

***Keywords- Cooperative, Cyclostationarity-Based, Low-Complexity Spectrum Sensing, Soft Combination.***

## I. INTRODUCTION

Traditional non-beneficial usage of available bandwidths, along with the increase in demand for higher data rates, leads to the development of novel techniques for flexible and efficient access to the licensed frequency bands. To avoid causing a destructive interference, secondary users (SUs) should reliably detect the presence of primary users (PUs). Therefore, the ability to perform reliable spectrum sensing is crucial to SUs. However, the signals received by cognitive radios usually are effected by the fading impairments of radio-frequency channels, and consequently, SUs should be able to detect very week signals in very low SNR situations [1].

A common detector that needs no prior knowledge about the PU signal is the well-known energy detector (ED). But, because of the so-called noise uncertainty drawback in EDs family [1], they cannot perform well in low-SNR conditions. However, in wireless networks there is usually some information about the modulation properties of the primary signal [2]. These properties could be exploited in the design of detectors that have acceptable performance in very low SNRs. One popular approach is the cyclostationary (CS) detection method [2, 3], which operates much better than energy detection, but is generally more complex [4]. These detectors can inherently distinguish PUs from SUs as well as interferers, if they exhibit dissimilar cyclic features. This important requirement could not be satisfied by conventional energy detectors [5, 6].

One of the key challenges in CS-based spectrum sensing is the computational complexity associated with CS detection algorithms. In fact, there is a trade-off among the complexity, the required sensing-time and the probability of detection of CS detector. The CS detector that recently has been proposed in the literature is the multi-cycle detector [7, 8], which is based on Dandawaté-Giannakis' algorithm [9]. But the main drawback of this approach is its complexity of implementation. To meet the sensing-time and complexity requirements, in this paper, we propose a novel multi-cycle sensing scheme by exploiting the second-order cyclostationarity of the OFDM-based primary signals. The proposed method drastically reduces the implementation complexity, while maintaining a comparable detection performance to multi-cycle method of [7].

In order to further alleviate the impact of shadowing and fading impairments, cooperative spectrum sensing (CSS) methods are proposed in the literature [10, 7]. In this paper, we propose a novel cooperative sensing method for fusing the test statistics is suggested. Also this method requires more communication bandwidth, but has better detection performance compared with existing methods.

This paper is organized as follows. In Section II, the system model is briefly introduced. Section III presents the proposed and the GLRT-based spectrum sensing methods. In addition, computational complexities of the proposed, GLRT-based and energy detection methods are compared. In section IV, cooperative spectrum sensing methods are proposed. Extensive simulation results are conducted in Section V. Finally, the conclusions are drawn in Section VI.

## II. System Model and Preliminaries

In this paper, we assume that CR receives the primary signal through Rayleigh fading channel. The baseband discrete-time model is given by:

$$x[n]=\eta hs[n]+w[n],\ \ n=1,\dots,M \qquad (1)$$

where $h:N_c(0,\sigma_h^2)$ and $w[n]:N_c(0,\sigma_w^2)$ are circularly symmetric complex Gaussian (CSCG) random processes, representing time-invariant frequency-nonselective Rayleigh fading exposed by the channel between PU and CR, and AWGN channel, respectively. In addition, $s[n]$ and $x[n]$ are respectively the PU signal and the received signal at CR. Moreover, $\eta=0$ and $\eta=1$ correspond to $H_0$ and $H_1$ hypotheses, respectively.

The process $x[n]$ is said to be (second-order) cyclostationary (in the wide sense) if its mean and autocorrelation functions ($R_{xx^*}(n;\nu)=E\{x[n]x^*[n+\nu]\}$) are periodic with some period $T$ [9], [11]Hence, the autocorrelation function can be represented by Fourier series expansion [11], and with the assumption of its convergence, we can write $R_{xx^*}(n;\nu)=\sum_{\alpha\in A}R_{xx^*}^{\alpha}(\nu)e^{j2\pi\alpha n}$ where $R_{xx^*}(n;\nu)$ is the conjugate cyclic auto-correlation function and the sum is taken over the integer multiples of fundamental frequencies (i.e. $A=\{m/T\}$, $m$ integer), called cycle frequencies $\alpha$. Therefore, the cyclic characteristics of autocorrelation function can be completely described by its Fourier coefficients. These coefficients, which depend on the lag parameter $\nu$, are called Cyclic Autocorrelation Function (CAF) and can be calculated as $R_{xx^*}^{\alpha}(\nu)=\lim_{M\to\infty}\frac{1}{M}\sum_{i=1}^{M}R_{xx^*}(i,\nu)e^{-j2\pi\alpha i}$ [11]. It has been proved that the fundamental cycle frequency of OFDM signal is the inverse of the useful OFDM symbol length [12], that is $1/T_s$, where $T_s=T_u+T_g$, $T_u$ and $T_g$ denote the useful symbol length and cyclic prefix length, respectively. Authors in [12] proved that the OFDM signal has strong cyclostationarity at time-lag $\nu=\pm T_u$.

## III. Non-Cooperative Cyclostationarity Detection

In this section, we derive some decision statistics for the considered hypothesis testing problem. A discrete-time unbiased and consistent estimation of the CAF of a random process $x[n]$ is given as [2]:

$$\hat{R}_{xx^*}^{\alpha}(\nu)=\frac{1}{M}\sum_{i=1}^{M}x[i]x^*[i+\nu]e^{-j2\pi\alpha i} \qquad (2)$$

In order to establish the decision statistic, we exploit some properties of the estimated CAF.

### A. Proposed Spectrum Sensing Method

In this section, we propose a low-complexity cyclostationarity detection statistic based on estimated CAF properties of OFDM signals. As we know, when the lag parameter of estimated CAF sets to $\nu=\pm T_u$, the CAF reveal local peaks at multiples of fundamental cycle frequency of OFDM signal, that is $k/T_u, k=\pm1,\pm2,\dots$ . At the other cycle frequencies, the magnitude of CAF should be small in compared with local peaks. This property is demonstrated in Fig. 1, where the signal is an IEEE 802.11a WLAN OFDM. The cross section view of Fig. 1 for constant $\nu=T_u$ is shown in Fig. 2. This figure reveals that the amplitude of peaks decrease as $k$ increases.

Based on mentioned property, we propose the following multi-cycle test statistic:

$$T=\frac{\sum_{k=-L}^{L}\left|R_{xx^*}^{k/T_u}(T_u)\right|^2}{\sum_{k=-L}^{L}\left|R_{xx^*}^{(k+\varepsilon_k)/T_u}(T_u)\right|^2} \qquad (3)$$

where $\varepsilon_k$ is chosen so that the $(k+\varepsilon_k)/T_u$ does not belong to the set of cycle frequencies $k/T_u$. For example, it can be an arbitrary (non-integer) number

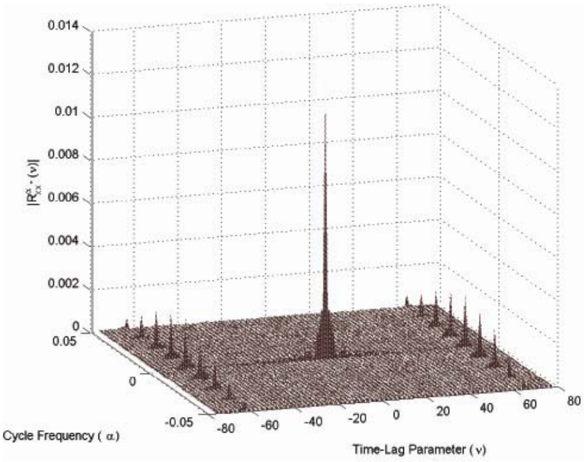


Fig 1. Three dimensional view of $\hat{R}^{\alpha}_{xx^*}$ for OFDM signal.

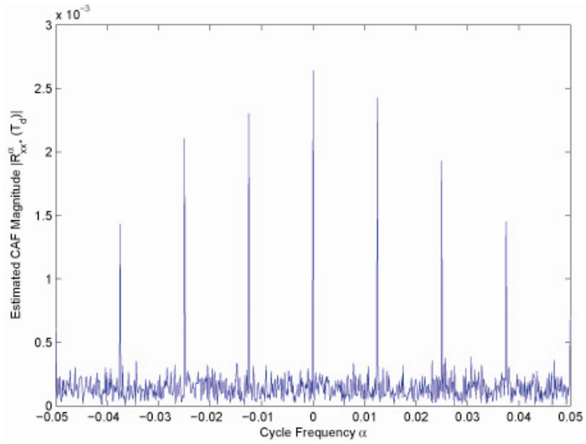


Fig 2. Magnitude of estimated CAF for OFDM signal, at the lag parameter $\nu = T_u$.

in the interval $(0.25, 0.75)$. When the signal is present, the denominator approaches zero, while the numerator increases. Hence, the statistic will be increased, and detector will decide $H_1$.

In order to perform binary hypothesis testing, we require the distribution of the test statistic under null hypothesis testing. Under $H_0$, we have $x[n] = w[n]$, where $w[n] : N_c(0, \sigma^2)$. Using the central limit theorem [13], we can obtain the asymptotic distribution of CAF under $H_0$. After some tedious but straightforward calculations, we get the following distribution under the null hypothesis (see Appendix A):

$$\hat{R}^{\alpha}_{ww^*}(\nu) : N_c\left(0, \frac{\sigma^4}{M}\right) \tag{4}$$

Therefore,

$$\frac{2M}{\sigma^4}\left|\hat{R}^{\alpha}_{ww^*}(\nu)\right|^2 : \chi_2^2 \tag{5}$$

Since the estimated CAFs, $\hat{R}^{\alpha_k}_{ww^*}(\nu)$, $k = 0, \pm 1, \pm 2, \ldots, \pm L$ are asymptotically independent under null hypothesis testing [7], distribution of $\frac{2M}{\sigma^4}\left|R^{k/T_u}_{xx^*}(T_u)\right|^2$ and $\frac{2M}{\sigma^4}\left|R^{(k+\varepsilon_k)/T_u}_{xx^*}(T_u)\right|^2$ are chi-square with $2$ degrees of freedom. Due to the fact that the sum of the independent chi-square random variable is also a chi-square random variable whose degrees of freedom is the sum of the degrees of freedom of independent random variables, $\frac{2M}{\sigma^4}\sum_{k=-L}^{L}\left|R^{k/T_u}_{xx^*}(T_u)\right|^2$ and $\frac{2M}{\sigma^4}\sum_{k=-L}^{L}\left|R^{(k+\varepsilon_k)/T_u}_{xx^*}(T_u)\right|^2$ are chi-square with $2(2L+1)$ degrees of freedom under the null hypothesis testing. Then the distribution of the test statistic under the null hypothesis testing is

$$T : F(2(2L+1), 2(2L+1)), \text{under} \mathrm{H}_0 \tag{6}$$

where $F(d_1, d_2)$ denotes the $F$ distribution, $d_1$ and $d_2$ are the numerator and denominator degrees of freedom, respectively. The pdf of a F distributed random variable is given by [14]

$$p(x) = \begin{cases} \dfrac{(d_1/d_2)^{\frac{d_1}{2}}}{B(\frac{d_1}{2}, \frac{d_2}{2})} \dfrac{x^{\frac{d_1}{2}-1}}{(1+\frac{d_1}{d_2}x)^{(d_1+d_2)/2}} & x > 0 \\ 0 & x < 0 \end{cases} \tag{7}$$

where $B(u, v)$ is Beta function

$$B(u, v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} \tag{8}$$

where $\Gamma(u)$ is Gamma function

$$\Gamma(u) = \int_0^{\infty} t^{u-1} e^{-t}\, dt. \tag{9}$$

Summarizing above discussions, we conclude the following low-complexity hypothesis test based on the second-order cyclostationary properties of OFDM signals:

$$\begin{aligned} &\text{(i) if } T < F^{-1}_{F(2(2L+1),2(2L+1))}\left(1-P_{fa}\right) \Rightarrow \text{decide} \mathrm{H}_0, \\ &\text{(ii) if } T \geq F^{-1}_{F(2(2L+1),2(2L+1))}\left(1-P_{fa}\right) \Rightarrow \text{decide} \mathrm{H}_1. \end{aligned} \tag{10}$$

where $F^{-1}_{F(d_1,d_2)}(x)$ denotes the inverse cumulative distribution function of $F(d_1, d_2)$ at point $x$. In Section V, we will evaluate the performance of the proposed detection method via simulation experiments.

*B. GLRT-based Cyclostationarity Sensing Method*

This subsection, briefly introduces the well-known Dandawaté-Giannakis's method [9] which is recently modified in [7]. This statistical test relies upon the asymptotic normality and consistency of second-order cyclic statistics, and detects the presence of cycles in second-order cyclic cumulates, without assuming any specific distribution on the transmitted data [9]. For more details, the reader is referred to [7, 9, 15].

Let we define a matrix consisting of CAF estimates at the cycle frequency $\alpha$ for different time lags:

$$\underline{r}^{\alpha}_{xx^*} = \Big[\Re\{\hat{R}^{\alpha}_{xx^*}(\nu_1)\},\ldots,\Re\{\hat{R}^{\alpha}_{xx^*}(\nu_N)\}, \Im\{\hat{R}^{\alpha}_{xx^*}(\nu_1)\},\ldots,\Im\{\hat{R}^{\alpha}_{xx^*}(\nu_N)\}\Big], \quad (11)$$

where $\Re\{\}$ and $\Im\{\}$ denote the real and imaginary parts, respectively. It has been shown that $\lim_{M\to\infty}\sqrt{M}\,\underline{r}^{\alpha}_{xx^*} \overset{D}{=} N(\underline{r}^{\alpha}_{xx^*},\Sigma_{xx^*})$, where $\overset{D}{=}$ denote the convergence in distribution and $N(\underline{\mu},\Sigma_{xx^*})$ is a multivariate normal distribution with mean vector $\underline{\mu}$ and covariance matrix $\Sigma_{xx^*}$ [9]. Noted that $\underline{r}^{\alpha}_{xx^*}$ is non-random, so the distribution of $\hat{\underline{r}}^{\alpha}_{xx^*}$ under $H_0$ and $H_1$ differs only in the mean.

The asymptotic complex normality of $\hat{\underline{r}}^{\alpha}_{xx^*}$ allows proposing the following generalized likelihood ratio, [7]

$$\Lambda_{GLR} = \frac{f\left(\hat{\underline{r}}^{\alpha}_{xx^*}\,|\,H_1\right)}{f\left(\hat{\underline{r}}^{\alpha}_{xx^*}\,|\,H_0\right)} = \exp\left(\frac{1}{2}M\hat{\underline{r}}^{\alpha}_{xx^*}\hat{\Sigma}^{-1}_{xx^*}(\hat{\underline{r}}^{\alpha}_{xx^*})^t\right), \quad (12)$$

where $\hat{\Sigma}_{xx^*}$ is an estimation of the asymptotic covariance matrix of $\hat{\underline{r}}^{\alpha}_{xx^*}$, defined as:

$$\hat{\Sigma}_{xx^*} = \begin{bmatrix} Re\left\{\frac{\mathbf{Q}+\mathbf{P}}{2}\right\} & Im\left\{\frac{\mathbf{Q}-\mathbf{P}}{2}\right\} \\ Im\left\{\frac{\mathbf{Q}+\mathbf{P}}{2}\right\} & Re\left\{\frac{\mathbf{P}-\mathbf{Q}}{2}\right\} \end{bmatrix}. \quad (13)$$

In the above equation, the two covariance matrices $\mathbf{Q}$ and $\mathbf{P}$ are by:

$$\mathbf{Q} = \frac{1}{MT}\sum_{s=-(T-1)/2}^{(T-1)/2} W(s) F_{\tau}(\alpha-\frac{2\pi s}{M})F_{\tau}(\alpha+\frac{2\pi s}{M})$$
$$\mathbf{P} = \frac{1}{MT}\sum_{s=-(T-1)/2}^{(T-1)/2} W(s) F^*_{\tau}(\alpha+\frac{2\pi s}{M})F_{\tau}(\alpha+\frac{2\pi s}{M}), \quad (14)$$

where $W$ is a normalized spectral window of odd length $T$ and $F_{\tau}(w) = \frac{1}{M}\sum_{k=1}^{M} x[k]x^*[k+\tau]e^{-jwk}$.

Finally, the generalized log-likelihood test statistic for the binary hypothesis testing corresponding to signal $x[n]$ is:

$$Z^{\alpha}_{x} = 2\ln(\Lambda_{GLR}) = M\hat{\underline{r}}^{\alpha}_{xx^*}.\hat{\Sigma}^{-1}_{xx^*}.(\hat{\underline{r}}^{\alpha}_{xx^*})^t, \quad (15)$$

where $t$ denotes the conjugate transpose of a matrix. To set a threshold for hypothesis testing, we need the asymptotic distribution of $Z^{\alpha}_{x}$. In [9], it is shown that, regardless of the distribution of the input data, the asymptotic distribution of the $Z^{\alpha}_{x}$ under the hypothesis $H_0$ is central chi-squared with $2N$ degrees of freedom (i.e. $\lim_{M\to\infty} Z^{\alpha}_{x} \overset{D}{=} \chi^2_{2N}$).

Furthermore, by detecting multiple cycle frequencies at the same time, $\{\alpha_j\}_{j=1}^{S}$, one can improve the performance of detection. For this end, we employ the following test statistics for CS feature detection problem [7]:

$$Z_{EGC} = \sum_{j=1}^{S} Z^{\alpha_j}_{x} \quad (16)$$

where $Z_{EGC}$ is the corresponding multi-cycle test statistic, and $S$ is the number of cycles that we are interested in to detect. The CAF estimates for different candidate cycle frequencies are independent under $H_0$ [7], so the asymptotic distribution of $Z_{EGC}$ is,

$$Z_{EGC|H_0} \sim \chi^2_{2NS} \quad (17)$$

Performance of the proposed scheme can be evaluated by calculating the probability of false alarm. In this case, the false alarm probability is calculated as bellow,

$$P_F = pr\{Z_{EGC} > z_{th}\,|H_0\} = 1 - F_{\chi^2_{2NS}}(z_{th}) = 1 - \frac{\gamma(NS, z_{th}/2)}{\Gamma(NS)} \quad (18)$$

Where $F_{\chi^2_r}(z)$, $\gamma(g,z)$ and $\Gamma(.)$ are the cumulative distribution function of a chi-squared random variable with $r$ degree of freedom at the point z, the lower incomplete Gamma function and the complete Gamma function, respectively (the last equality can be found in [16]).

Therefore the detection threshold is $z_{th} = F_{\chi^2_{2NS}}(1-P_f)$, and the resultant binary hypohesis test can be formulated as

$$\text{(i) if } Z_{EGC} < z_{th} \quad \Rightarrow \text{decide} H_0,$$
$$\text{(ii) if } Z_{EGC} \geq z_{th} \quad \Rightarrow \text{decide} H_1.$$

### C. *Discussion on Computational Complexity*

In this section, we compare the computational complexities of the proposed, GLRT-based and

Table 1. Comparison between approximated computational complexities of the proposed, GLRT-based and energy detection methods

| Detection Method | Computational complexity | Ex. $M=4096$ $L=4$ $T=2049$ |
|---|---|---|
| Proposed | $M(1+\frac{1}{2}\log_2 M)$ | 28672 |
| GLRT-Based | $M(1+\frac{1}{2}\log_2 M)$ $+4(2L+1)T$ | 28672 +73764 |
| Energy | $M$ | 4096 |

energy detection methods. The most time-consuming step in the GLRT-based algorithm is the estimation of the covariance matrix (13). Since this step is not needed in the proposed algorithm, it causes a significant reduction in computational complexity. As it can be seen in table. I, the computational complexity of the proposed method can be expressed as $M(1+(1/2)\log_2 M)$, where $M$ denotes the number of observation samples. In the above expression, it is assumed that the fast-Fourier transform (FFT) algorithm is used for computing the CAF and $F_\tau(\omega)$. The term $(M/2)\log_2 M$ is for the computation of FFT and $M$ denotes the number of multiplications needed in CAF.

However, the computational complexity of GLRT-based method for one time-lag ($N=1$) is given as $M(1+(1/2)\log_2 M)+4(2L+1)T$. The term $M(1+(1/2)\log_2 M)$ is for computation of CAF and the term $4(2L+1)T$ is due to the estimation of the covariance matrix. Note that the parameter $T$ is the length of the normalized window used in estimation of the covariance matrix. The parameter $2L+1$ is the number of cycle frequencies that used in the test statistic.

It should be noted that in the proposed method, increasing the number of cycle frequencies employed in the decision statistic does not significantly increase the computational complexity.

As a final remark, the computational complexity of the energy detection method can be expressed by $M$. In energy detection method, simply the autocorrelation of the signal is considered. Although this method has very low complexity, however, it has some challenging drawbacks (see section V-B).

In order to palpable difference among computational complexity of this three methods, we get a practical example in third column of table. 1.

There is a trade-off between computational complexity and performance. The performance of the proposed method is better than the energy detection and a bit worse than the GLRT-based method while computational complexity of the proposed method is $4(2L+1)T$ lower than the GLRT-based method and $(M/2)\log_2 M$ higher than the energy detection.

## IV. COOPERATIVE SPECTRUM SENSING METHODS

One of the most challenging issues in the secondary access of CRs to the licensed bands is the reliable detection primary users in low SNR conditions. To overcome this problem, cooperative spectrum sensing (CSS) methods are proposed in the literature. In this section, some CSS methods based on the proposed low-complexity cyclostationary detection is presented. It is assumed that each CR observes $M$ samples and then forms the proposed decision statistic (3). We assume that the secondary users are independent given $H_0$ or $H_1$ [7].

### A. Conventional Cooperative Spectrum Sensing Methods

In centralized cooperative spectrum sensing, the fusion center decides about the presence/absence of a PU based on the decision statistics received from CRs. Different methods are proposed in literature for fusing the decisions statistics transmitted from CRs. A simple fusion rule is the summation of test statistics:

$$T_s = \sum_{n=1}^{S} T_n \tag{19}$$

The CDF of $T_s$ can be obtained by numerical simulations or convolving the pdfs of different test statistics $T_n$. However, we propose a simple but efficient threshold selection method in Appendix B.

Another well-known fusion rule is the MAX rule:

$$T_m = \max_{n=1,\dots,S} T_n \tag{20}$$

The null distribution of the above test statistic is computed in Appendix C.

### B. Proposed Cooperative Spectrum Sensing Methods

In what follow, we propose a new method for the cooperative sensing. Suppose that each CR calculates the numerator and denominator of (3), separately. That is, for $n$ th CR we have

$$T_{nom,n} = \sum_{k=-L}^{L} \left| R_{xx^*}^{k/T_u}(T_u) \right|^2, \tag{21}$$

and

$$T_{den,n} = \sum_{k=-L}^{L} \left| R_{xx^*}^{(k+\varepsilon_k)/T_u}(T_u) \right|^2. \tag{22}$$

After that (20) and (21) are computed at each CR, they are transmitted to the fusion center. Then, we constitute the following decision statistic at the fusion center:

$$T_{proposed} = \frac{\sum_{n=1}^{S} T_{nom,n}}{\sum_{n=1}^{S} T_{den,n}} \tag{23}$$

The null distributions of $T_{nom,n}$ and $T_{den,n}$ is chi-square with $2(2L+1)$ degrees of freedom. Thus, $\sum_{n=1}^{S} T_{nom,n}$ and $\sum_{n=1}^{S} T_{den,n}$ are distributed as $\chi^2_{2(2L+1)S}$.

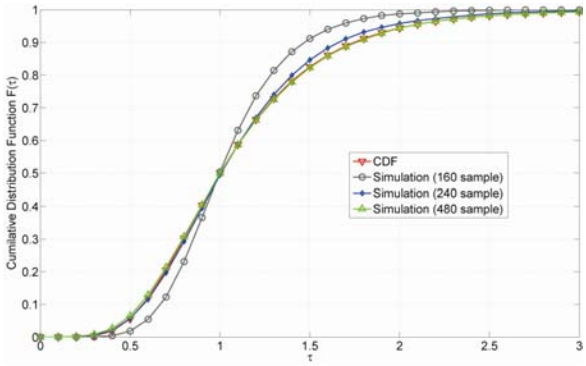


Fig 3. Theoretical versus simulated CDF of the proposed decision statistic under null hypothesis. Increasing the detection time improves the accuracy of the asymptotic distribution.

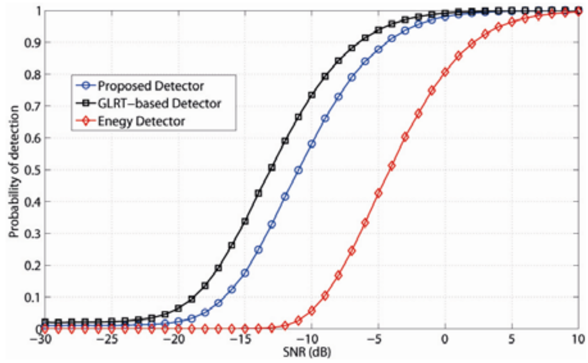


Fig 5. Probability of detection for the proposed, GLRT-based and energy detection methods in a Rayleigh flat fading channel. The sensing time is set to 1.6 ms (=32,000 sample).

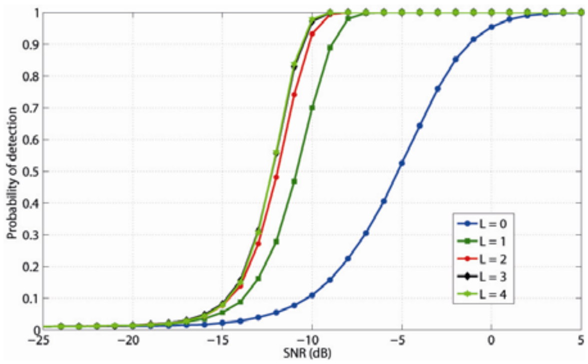


Fig 4. Probability of detection versus SNR for different numbers of cycle frequencies. The sensing time is set to 1.6 ms (=32,000 sample).

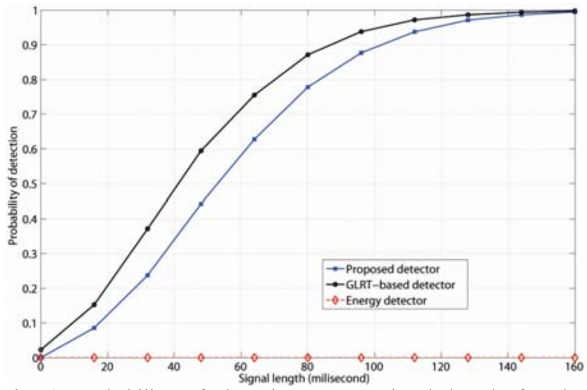


Fig 6. Probability of detection versus signal length for the proposed, GLRT-based and energy detection methods (AWGN channel with -20 dB).

Therefore, the null distribution of the proposed test statistic can be obtained as:

$$T_{proposed} : F(2(2L+1)S, 2(2L+1)S). \qquad (24)$$

Although the overhead of this method is twice the (18) and (19), but it will be shown that it has better performance than the others. Furthermore, since its null distribution has close-form expression, the threshold selection at the FC is straightforward.

## V. SIMULATION RESULTS

In this paper assumes that the primary user signal is OFDM signal. The baseband OFDM signal is given by [3]

$$x(t) = \frac{1}{\sqrt{N_c}} \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N_c} d_{n,k} e^{j2\pi k \Delta f (t - nT_s - \varepsilon)} \times g_r(t - nT_s - \varepsilon) e^{-j2\pi((N_c-1)/2)\Delta f t} \qquad (25)$$

where $d_{n,k}$ is the $n$ th information symbol modulated on the $k$ th carrier, $N_c$ number of carrier, $\Delta f$ the carrier separation, $\varepsilon$ the unknown symbol timing and $g_r(t)$ the rectangular pulse function of length $T_s$.

In all of the simulations, the primary network is an IEEE 802.11 WLAN network. The number of subcarriers $N_{FFT} = 64$ of which $N_{occ} = 52$ are occupied and the cyclic prefix length $N_g = 16$. The subcarrier modulation of OFDM signal is QPSK. The symbol rate and sampling rate are 250 KSym/Sec and $R_s$ =20 MHz, respectively. In all simulations, we employ the false-alarm rate of $0.01$ and the time-lag of $\nu = T_u$ (the value of $T_u$ can be computed as $T_u = N_{FFT}/R_s$ ). Cyclostationary features of an OFDM signal occurs in multiples of the symbol rate $\alpha = k/T_s, k = 0, \pm 1, \pm 2, \ldots$. Prior knowledge of $T_g$ (cyclic prefix duration) and $T_u$ is assumed. The local peaks of the estimated cyclic autocorrelation function $\hat{R}^{\alpha}_{xx^*}(\nu)$ are occurred in $\nu = \pm T_u$ and $\alpha = k/T_s$. In this paper, a kaiser window length and $\beta$ parameter are set to $2049$ and $10$ , respectively. The parameters employed for approximating the CDF are $H = 1000$ and $\eta = 0.5$. In this section, the plotted simulation curve are averages over 10,000 experiment.

### A. *Investigating the Asymptotic Distribution of the Proposed Decision Statistic*

The accuracy of the asymptotic distribution (6) under the null hypothesis is evaluated in Fig. 3. The

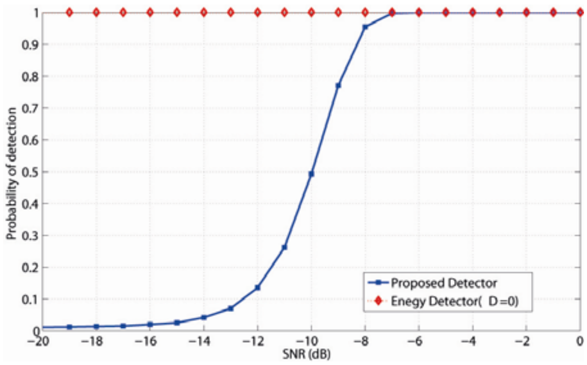


Fig 7. Probability of detection versus SNR for the proposed, GLRT-based and energy detector methods over AWGN environments and in presence of interference. The sensing time is set to 1.6 ms (=32,000 sample).

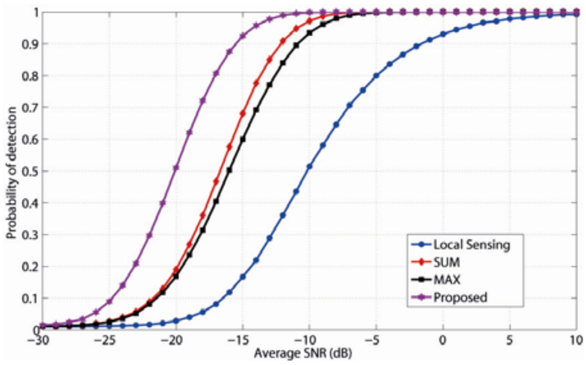


Fig 8. Probability of detection versus channel average SNR for different proposed soft cooperative methods over Rayleigh flat fading channels. The vector of local channel SNRs (in dB) at secondary users is defined as $\gamma$=[$\bar{\gamma}-5, \bar{\gamma}-4, \bar{\gamma}-3, \bar{\gamma}-2, \bar{\gamma}-1, \bar{\gamma}+1, \bar{\gamma}+2, \bar{\gamma}+3, \bar{\gamma}+4, \bar{\gamma}+5$] where $\bar{\gamma}$ is average SNR. The sensing time is set to 1.6 ms (=32,000 sample).

theoretical CDF curve is obtained from equation (3) with $L=5$. The PU signals are assumed to be white Gaussian random variables. It is evident that if the detection time is sufficiently long, the simulation results should confirm the theoretical asymptotic distribution curves.

### B. *Detection Performance Comparisons*

In this subsection, we investigate the performance of the proposed method and further compare it with the GLRT-based and energy detection methods. Fig. 4 depicts the probability of detection versus SNR for different numbers of cycle frequencies. It is evident that as the number of cycle frequencies increases, the performance improves.

In Fig. 5 we compare the detection performance of the three methods as a function of channel SNR. The energy detector is assumed to experiences 1 dB noise uncertainty. As mentioned in section III-C, increasing the number of cycle frequencies does not significantly heighten the computational complexity of the proposed method. Therefore, we use $9$ cycle frequencies in detection process of the proposed and GLRT-based methods. The results reveal that while

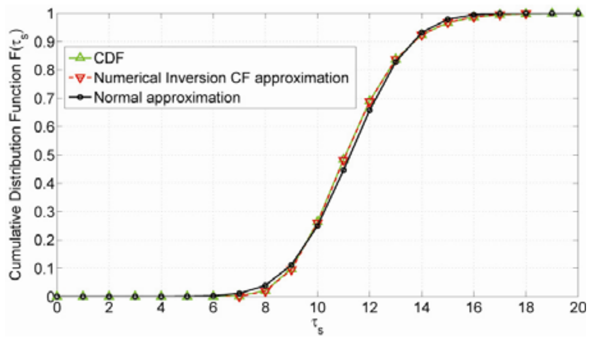


Fig 9. Accuracy comparison of proposed Gaussian approximation and numerical inversion method for white Gaussian noise. The sensing time is set to 1.6 ms (10 CRs in network).

our proposed method has much lower complexity than the GLRT-based method, it provides a comparable performance. Moreover, it has better detection performance than the ED method.

Fig. 6 demonstrates the detection probability versus signal length for $SNR$=-20 dB. As we can see, increasing the sensing duration does not necessarily improve the performance of the energy detector (because of the so-called SNR-wall impact [17]).

Fig. 7 depicts the performance of the proposed method in presence of an interfering signal. The interfering signal is an OFDM signal with 4-QAM subcarrier modulation ( $N_{FFT}=200, N_{occ}=200, N_g=16$ ). The SNR of the interference is $-5$ dB. As it is evident, the energy detection cannot distinguish between a weak interfering signal and a PU signal. Furthermore, it cannot discriminate between primary users belonging to different primary networks with different transmission technologies.

Fig. 8 depicts the performances of the soft cooperative sensing methods. Each employs $2$ cycle frequencies for the detector proposed in (3). As we can see, the proposed cooperative method will have better detection performance compared to the other methods, at the expense of an increase in communication overhead of the secondary network. Also, the SUM method has better performance than the MAX method.

The accuracy of the approximated CDF proposed in (32), approximation with normal distribution, and simulated CDF is investigated for decision statistic (19) in Fig. 9. It is assumed that each CR calculates the decision statistic with 9 cycle frequencies. As it can be seen, the numerical inversion method is accurate, while the Gaussian approximation performs with lower accuracy.

## VI. CONCLUSIONS

In this paper, a fast reduced-complexity multi-cycle cyclostationariy detector has been proposed and then its performance and computational complexity has been compared with well-known multi-cycle GLRT-based detector as well as energy detector. The analytical and simulation results have been shown that the computational complexity has been significantly reduced, while a slight degradation in detection performance is occurred, compared to the GLRT-based scheme. Also, we have shown that the proposed method is robust against the noise uncertainty problem, in contrast to the energy detection methods. The performance of the proposed method is better than the energy detection and a bit worse than the GLRT-based method while computational complexity of the proposed method is much lower than the GLRT-based method and higher than the energy detection.

Different cooperative spectrum sensing methods has been analyzed in this paper. We further propose a cooperative detector that has better detection performance than the existing methods, at a cost of a little increase in communication overhead of the cognitive radio network. Finally, we have proposed a straightforward method for threshold selection at the fusion center. Particularly, we have formulated a general approach for calculating the null distribution of the decision statistic of the cooperative detectors.

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## APPENDIX A
## DISTRIBUTION OF ESTIMATED CAF

The circularly symmetric Gaussian noise process $w[n]$ can be represented by its real and imaginary parts as $w[n] = w_r[n] + jw_i[n]$, where each one is distributed as $N(0, \sigma^2_{xx^*}/2)$. Therefore we can write:

$$\begin{aligned}\hat{R}^{\alpha}_{ww^*}(\tau) &= \frac{1}{M}\sum_{n=1}^{M} w[n]w^*[n+v]e^{-j2\pi n\alpha} \\ &= \frac{1}{M}\sum_{n=1}^{M}(w_r[n]+jw_i[n]) \\ &\quad (w_r[n+v]-jw_i[n+v])e^{-j2\pi n\alpha} \\ &= Re\{\hat{R}^{\alpha}_{ww^*}(\tau)\} + jIm\{\hat{R}^{\alpha}_{ww^*}(\tau)\}\end{aligned} \tag{26}$$

In following, we will obtain the distribution of the real part. The distribution of the imaginary part can be similarly calculated. We can write

$$\begin{aligned}Re\{\hat{R}^{\alpha}_{ww^*}(\tau)\} &= \frac{1}{M}\sum_{n=1}^{M} w_r[n]w_r[n+v]cos(2\pi n\alpha) \\ &-\frac{1}{M}\sum_{n=1}^{M} w_r[n]w_i[n+v]sin(2\pi n\alpha) \\ &-\frac{1}{M}\sum_{n=1}^{M} w_i[n]w_r[n+v]sin(2\pi n\alpha) \\ &-\frac{1}{M}\sum_{n=1}^{M} w_i[n]w_i[n+v]cos(2\pi n\alpha)\end{aligned} \tag{27}$$

Using CLT, we have $Re\{\hat{R}^{\alpha}_{ww^*}(\tau)\} \sim N(0, Var)$. The variance is calculated as

$$\begin{aligned}Var &= \frac{\sigma^4}{4M^2}\left(\sum_{n=1}^{M} cos^2(2\pi n\alpha) \right. \\ &\quad \left. + sin^2(2\pi n\alpha)\right) \times 2 \\ &= \frac{\sigma^4}{2M}\end{aligned} \tag{28}$$

Similarly we can see $Re\{\hat{R}^{\alpha}_{ww^*}(\tau)\} \sim N(0, \frac{\sigma^4}{2M})$

Therefore, $\hat{R}^{\alpha}_{ww^*}(\tau) \sim N_c(0, \sigma^4/M)$.

## APPENDIX B
## ESTIMATING THE NULL DISTRIBUTION OF SUM FUSION RULE

Here, we propose two methods for approximating the null distribution of (19). With the assumption of 10 or more CRs in network and by using CLT [14], we propose to approximate the distribution of (18) with a Gaussian pdf:

$$T_s \sim N\left(\sum_{k=1}^{S}\mu_k, \sum_{k=1}^{S}\sigma_k^2\right), \tag{29}$$

where $\mu_k$ and $\sigma_k^2$ are mean and variance of $k$ th SU's test statistic, respectively. Mean and variance of an $F$ random variable with $d_1$ and $d_2$ degrees of freedoms can be computed as [14]:

$$\mu = \frac{d_2}{d_2 - 2} \qquad d_2 > 2 \tag{30}$$

$$\sigma^2 = \frac{2d_2^2(d_1 + d_2 - 2)}{d_1(d_2-2)^2(d_2-4)} \quad d_2 > 4 \tag{31}$$

Alternatively, we propose the following more complex, but more accurate, method for approximating the null distribution. In fact, we propose to numerically invert the resulting characteristic function based on the method presented in [18]. Using the same line as [18], the cumulative distribution function $F(y)$ of random variable $Y$ with zero mean and unit variance can be approximated by:

$$F(y) \approx \frac{1}{2} + \frac{\eta y}{2\pi} - \sum_{\substack{v=1-H \\ v \neq 0}}^{H-1} \frac{\Phi_Y(\eta v)}{2\pi jv} e^{-j\eta v y} \tag{32}$$

where $\Phi_Y(.)$ is the characteristic function of Y. $\eta$ is a constant chosen such that the full range of distribution is represented. Furthermore, the characteristic function of a normalized variable $Z=(Y-\mu)/\sigma$ is given by $\Phi_Z(w)=\Phi_Y(\frac{w}{\sigma})\exp(\frac{-jw\mu}{\sigma})$.

Since (31) is defined for normalized random variable with zero mean and unit variance, the test statistic has to be normalized as well. We employ (30) and (31) expressions for computing the approximate CDF (32). On the other hand, the characteristic function of an $F$ random variable with $b_1$ and $b_2$ degrees of freedoms is defined as:

$$\Phi_{T_n}(w)=\frac{\Gamma(b_2)}{\Gamma(b_1)\Gamma(b_2-b_1)}\int_0^1 t^{b_1-1}(1-t)^{b_2-b_1-1}e^{xt}\,dt \tag{33}$$

Finally, the characteristic function of $T_s$ can be obtained as:

$$\Phi_{T_s}(w)=\prod_{n=1}^{S}\Phi_{T_n}(w) \tag{34}$$

APPENDIX C
THE DISTRIBUTION OF THE MAXIMUM OF S INDEPENDENT F RANDOM VARIABLE

Suppose $S$ independent random variables, each of them has an $F$ distribution with $d_{1,k}$ and $d_{2,k}$ degrees of freedom ( $k=1,2,\ldots,S$ ) for its nominator and denominator, respectively. The cumulative distribution function of an $F$ random variable with $d_1$ and $d_2$ degrees of freedom is given by:

$$F_F(T,d_1,d_2)=I_{\frac{d_1T}{d_1T+d_2}}(d_1/2,d_2/2), \tag{34}$$

where $I_x(a,b)$ is a regularized incomplete Beta function defined as $I_x(a,b)=B(x;a,b)/B(a,b)$. In this equation, $B(x;a,b)$ is an incomplete Beta function with $B(x;a,b)=\int_0^x t^{a-1}(1-t)^{b-1}dt$ . The cumulative distribution function of the maximum of S independent random variable is the product of the CDFs of the individual random variables.

$$p(\max_k T_k<a)=p(T_1<a,\ldots,T_S<a)$$
$$=\prod_{k=1}^{S}p(T_k<a) \tag{35}$$

Hence, CDF of the maximum of $S$ random variables with $d_{1,k}$ and $d_{2,k}$ numerator and denominator degrees of freedom for the $k$ th random variable, is given by

$$F_F(T_m,S,\{d_{1,k},d_{2,k}\}_{k=1}^{S})=\prod_{k=1}^{S}\left(I_{\frac{d_{1,k}T_k}{d_{1,k}T_k+d_{2,k}}}(d_{1,k}/2,d_{2,k}/2)\right). \tag{36}$$

We use the above equation for computing the intended CDF.

## REFERENCES


[1] R. Tandra and A. Sahai, "Fundamental limits on detection in low snr under noise uncertainty," in Proc. Wireless Commun. Symp. Signal Process., 2005, pp. 464–469.

[2] N.Han,S.H.Shon,J.H.Chung,andJ.M.Kim,"Spectralcor- relation based signal detection method for spectrum sensing in ieee 802.22 wran systems," in 8th Int. Conf. on Advanced Communication Technology (ICACT), 2006, pp. 1765–1770.

[3] M. Oner and F. Jondral, "Cyclostationarity-based methods for th extraction of the channel allocation information in a spectrum pooling system," in In Proc. IEEE Radio and Wireless Conf., 2004, pp. 279–282, atlanta, Georgia, USA.

[4] J. Ma, G. Y. Li, and B. H. Juang, "Signal processing in cognitive radio," Proc. IEEE, vol. 97, no. 5, pp. 805–823, 2009.

[5] E. Hossain and V. K. Bhargava, Cognitive Wireless Communication Networks. Springer US, 2007.

[6] H. Arslan, Cognitive Radio, Software De¿ned Radio, and Adaptive Wireless Systems. Springer, 2007.

[7] J. Lunden, V. Koivunen, A. Huttunen, and H. V. Poor, "Collaborative cyclostationary spectrum sensing for cognitive radio systems," IEEE Trans. Signal Process., vol. 57, no. 11, pp.4182–4195, Sep. 2009.

[8] ——, "Spectrum sensing in cognitive radios based on multiple cyclic frequencies," in Proc. 2nd Int. Conf. Cogn. Radio Oriented Wireless Netw. Commun. (CrownCom), Orlando, FL,Jul. 31- Aug. 3, 2007.

[9] A. V. Dandawat´e and G. B. Giannakis, "Statistical tests for presence of cyclostationarity," IEEE Trans. Signal Process., vol. 42, no. 9, pp. 2355–2369, Sep. 1994.

[10] K. B. Letaief and W. Zhang, "Cooperative communications for cognitive radio networks," Proc. IEEE, vol. 97, no. 5, pp. 878–893, May 2009.

[11] W. A. Gardner, A. Napolitano, and L. Paura, "Cyclostationarity: Half a century of research," Signal Process., vol. 86, pp.639–697, Apr. 2006.

[12] [12] M. Oner and F. Jondral, "Air interface identi¿cation for software radio systems," Int. J. Electron. Commun. (AEU), vol. 61, no. 2, pp. 104–117, Feb. 2007.

[13] S.M.Kay, Fundamentals of Statistical Signal Processing, vol. I: Estimation Theory. Prentice Hall, 1993.

[14] ——, Fundamentals of Statistical Signal Processing, vol. II: Detection Theory. Prentice Hall, 1998.

[15] A. V. Dandawat´e and G. B. Giannakis, "Asymptotic theory of mixed time averages and kth-order cyclic-moment and cumulant statistics," IEEE Trans. Inf. Theory, vol. 41, no. 1,pp. 216–232, Jan. 1995.

[16] C. Sun, W. Zhang, and K. B. Letaief, "Cooperative spectrum sensing for cognitive radios under bandwidth constraint," in Proc. IEEE Int. Wireless Commun. Networking Conf. (WCNC 2007), Mar. 2007, pp. 1–5.

[17] R. Tandra and A. Sahai, "Snr walls for signal detection," Selected Topics in Signal Processing, IEEE Journal of, vol. 2, no. 1, pp. 4 –17, feb. 2008.

[18] H.Bohman, "Numerical inversions of characteristic function," Scand. Actuarial J., pp. 121–124, 1975.

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