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# *A New Fractional Adaptive Filtering Method and its Application in Speech Enhancement*

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**Abstract**— In this paper, a modified version of an adaptive filtering technique, called fractional affine projection algorithm, is proposed for the dual-channel speech enhancement problem. The new adaptive filtering approach uses the fractional derivative together with the conventional first-order derivative of the mean-square error in its update equation. The update rule shows nonlinear behavior of step-size with respect to input signal. The proposed method is compared with the conventional methods of the least-mean-squares, normalized least-mean-squares, fractional least-mean-squares, normalized fractional least-mean-squares, and affine projection algorithms, both subjectively and objectively. The quality of noisy speech processed by applying different algorithms is evaluated objectively through the SNR and PESQ test measurements, and subjectively by conducting listening tests. Experimental results show that the fractional affine projection algorithm outperforms the conventional adaptive filtering methods in the sense of mean-square-error and quality of enhanced speech.

**Keywords**- *Speech Enhancement; Dual Channel speech Enhancement; Adaptive Filtering; Fractional Signal Processing; Affine Projection Algorithms.*

## I. INTRODUCTION

As many communication signals, the quality of speech signal will be affected by many phenomena during transmission of signal through the acoustic communication channel. The result of this affection will be unpleasant and will cause the quality reduction and intelligibility impairing of speech signal. To reduce the deficiency of acoustic noise on speech signal, many enhancement methods have been proposed [1]. The adaptive filtering methods are among common techniques which are employed in speech enhancement systems [2]. An example of such systems is the dual-channel speech enhancement, which uses two microphones for capturing contaminated speech and noise signal.

So far, many gradient-based algorithms have been proposed. The Least-Mean-Squares (LMS) algorithm is one of the common algorithms used [3]. The normalized version of LMS (i.e., NLMS) outperforms the LMS algorithm in the sense of stability. The Affine Projection Algorithm (APA) is obtained by generalization and modification of the NLMS algorithm [4]. Other adaptive filtering techniques such as Recursive Least-Squares (RLS) are also used for speech enhancement [5].

The concept of fractional-order operators have been investigated extensively in recent years in various signal processing theories and techniques [6]–[9]. Recently, a new adaptive LMS-based algorithm which is called Fractional LMS (FLMS) has been

proposed in some signal processing applications such as system identification [10]-[11]. FLMS employs the fractional-order derivative together with the conventional first-order derivative of mean-square error in its structure. It has also been shown that the FLMS algorithm outperforms LMS in speech enhancement. Furthermore, by normalization of the FLMS algorithm, a new adaptive filtering technique, called Normalized Fractional LMS (NFLMS), has been proposed for the enhancement of speech signals corrupted by noise [12].

In this paper, we propose a modified adaptive filtering method for dual-channel speech enhancement which is called Fractional Affine Projection Algorithm (FAPA). The proposed method has two main features. First, it exploits the benefits of APA, which is known to have a better approximation of the conventional recursive Newton method [13]. Moreover, it has been shown that the Affine projection algorithm has better convergence rate than LMS [14]. Second, the proposed method employs fractional derivatives in the definition of its update rule to improve the convergence performance of the conventional Affine projection.

The organization of this paper is as follows: Section 2 describes the structure of the dual-channel speech enhancement system together with the techniques of LMS, NLMS, FLMS, NFLMS, and APA. In Section 3, our proposed algorithm (i.e., FAPA) is introduced. Section 4 presents the experimental results and the comparisons made with the traditional adaptive filtering methods which are used in the context of speech enhancement. Concluding remarks are given in Section 5.

## II. BACKGROUND

### A. Speech Enhancement

Fig. 1 shows the block diagram for a general two-channel enhancement system. The clean speech signal  $s(n)$  is assumed to be present in only one channel, which is then corrupted by background noise  $b(n)$  to generate the noisy speech signal  $d(n)$ . The second channel has the reference noise signal  $u(n)$ . The adaptive filter  $W(z)$  tries to model the acoustic path transfer function  $P(z)$ . As a result, the filter output  $y(n)$  becomes an estimate of only noise present in  $d(n)$ . Finally, the output of the structure  $e(n)$  will be an estimate of the clean speech signal  $s(n)$ .

The output of the adaptive filter is given by:

$$y(n) = \mathbf{w}^T \mathbf{u}(n), \tag{1}$$

where  $\mathbf{w}$  is the weight vector. The Wiener-Hopf solution  $\mathbf{w}^{opt} = \mathbf{R}_u^{-1} \mathbf{r}_{du}$  gives the optimal weight vector for Eq. (1) by minimizing mean-square-errors of the following cost function:

$$J = E \left\| d - \mathbf{w}^T \mathbf{u} \right\|^2. \tag{2}$$

However, this solution introduces a high order of computational complexity. A simple recursive solution to the classical Wiener filtering problem is given by gradient-based algorithms such as the steepest-descent and regularized Newton techniques.

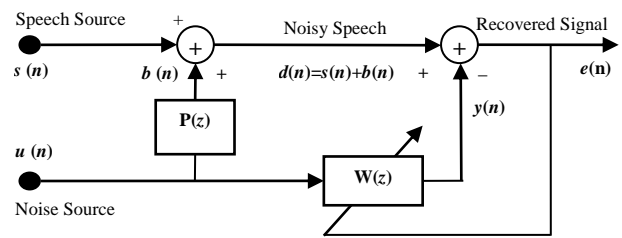


Figure 1: Dual-channel speech enhancement structure.

In the steepest-descent optimization method, the weight vector is made to evolve in the direction of negative gradient:

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) + \frac{\mu}{2} \left[ -\nabla \left( E \{ e^2(n) \} \right) \right], \\ &= \mathbf{w}(n) + \mu \left[ \mathbf{r}_{du} - \mathbf{R}_{uu} \mathbf{w}(n) \right]. \end{aligned} \tag{3}$$

Here,  $e(n)$  is the error signal,  $\mu$  is the step size,  $\mathbf{r}_{du}$  is the cross-correlation vector, and  $\mathbf{R}_{uu}$  is the correlation matrix.

In the Newton recursion method, the second derivative of mean-square-error is used for adaptation:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \left( \mathbf{R}_{uu} + \varepsilon \mathbf{I} \right)^{-1} \left[ \mathbf{r}_{du} - \mathbf{R}_{uu} \mathbf{w}(n) \right], \tag{4}$$

where  $\varepsilon$  is the regularization parameter.

### B. LMS Algorithm

The LMS algorithm is basically a simplification of steepest descent method, in which the gradient vector is estimated from available data when we operate in unknown environment [5].

To develop an estimate of the gradient vector, the most obvious strategy is to substitute estimates of the correlation matrix  $\mathbf{R}_{uu}$  and cross-correlation vector  $\mathbf{r}_{du}$  in the steepest descent update Eq. (3). The instantaneous estimates of  $\mathbf{R}_{uu}$  and  $\mathbf{r}_{du}$  are given respectively as:

$$\hat{\mathbf{R}}_{uu}(n) = \mathbf{u}(n) \mathbf{u}^T(n), \tag{5}$$

and

$$\hat{\mathbf{r}}_{du}(n) = d(n) \mathbf{u}(n). \tag{6}$$

Substituting these estimates in the steepest-descent algorithm, we get the following update rule for the tap-weight vectors:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) \left( d(n) - \mathbf{u}^T(n) \mathbf{w}(n) \right). \tag{7}$$

By defining the error signal:

$$e(n) = d(n) - y(n), \tag{8}$$

the final update formula for the tap weights is given as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \mathbf{u}(n) e(n). \tag{9}$$

### C. NLMS Algorithm

The adjustment applied to the tap-weight vector in LMS update rule is directly proportional to the tap-



input vector,  $\mathbf{u}(n)$ . Therefore, when  $\mathbf{u}(n)$  is large, LMS filters suffer from a gradient noise amplification problem. To overcome this difficulty, we may use the normalized LMS (NLMS) filter [13].

In structural terms, the NLMS filter is exactly the same as the standard LMS filter, but differs only in the way in which the weights are updated. The normalized LMS filter is manifestation of the principle of minimum disturbance [5]. From one iteration to the next, the weight vector of the adaptive filter should be changed in minimal manner, subject to a minimum constraint imposed on updated filter's output. The tap-weight adaptation rule is given by:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{\delta + \|\mathbf{u}(n)\|^2} \mathbf{u}(n)e(n), \quad (10)$$

where  $\|\mathbf{u}(n)\|^2$  is the power of input vector and  $\delta > 0$  is a constant factor.

#### D. Fractional LMS

In LMS, the weights are optimized in a manner that the error is minimized in mean-square sense. However, except in very special cases, the LMS algorithm is in general a sub-optimal technique [5]. In the FLMS, the weight update equation for the  $k^{th}$ -element, having only the first derivative term, is given by:

$$w_k(n+1) = w_k(n) - \mu \frac{\partial J(n)}{\partial w_k}, \quad (11)$$

$$(k = 0, 1, 2, \dots, M-1)$$

where  $M$  is the number of tap weights and  $n$  is the current time index. In deriving the fractional LMS (FLMS) algorithm, we have to use fractional derivatives in addition to the first derivative. The update relation for the  $k^{th}$ -element of the weight vector in FLMS is given by [10]:

$$w_k(n+1) = w_k(n) - \mu_1 \frac{\partial J(n)}{\partial w_k} - \mu_f \frac{\partial^\nu J(n)}{\partial w_k^\nu}, \quad (12)$$

where  $\nu$  ( $0 < \nu < 1$ ) is a real number,  $\mu_1$  is the first-order step size, and  $\mu_f$  is the fractional step-size.

The cost function given in Eq. (2) can be expanded in the following manner:

$$J(n) = d^2(n) - 2d(n) \sum_{i=0}^{M-1} w_i(n)u(n-i) + \sum_{i=0}^{M-1} w_i(n)u(n-i) \sum_{j=0}^{M-1} w_j(n)u(n-j). \quad (13)$$

By taking the  $\nu^{th}$ -order fractional derivative of the above cost function, we obtain

$$\frac{\partial^\nu J(n)}{\partial w_k^\nu} = -2(e(n)u(n-k))D^\nu w_k(n), \quad (14)$$

where  $D^\nu = \frac{\partial^\nu}{\partial w_k^\nu}$  is the Riemann-Liouville differential operator, which is defined

for  $\alpha \in \mathfrak{R}_+$  and  $n = \lceil \alpha \rceil$  with lower terminal at zero as follows [10]:

$$D^\alpha f(x) = \frac{1}{\Gamma(n-\alpha)} \left( \frac{d}{dx} \right)^n \int_0^x (x-t)^{n-\alpha-1} f(t) dt. \quad (15)$$

The fractional derivative with the order  $\alpha$  of the power function  $x^p$  can be written as:

$$D^\alpha x^p = \frac{\Gamma(p+1)}{\Gamma(p-\alpha+1)} x^{p-\alpha}. \quad (16)$$

By applying the above operator to Eq. (14), we obtain

$$\frac{\partial^\nu J(n)}{\partial w_k^\nu} = -2(e(n)u(n-k)) \frac{1}{\Gamma(2-\nu)} w_k^{1-\nu}(n). \quad (17)$$

After substituting the derivative terms in Eq. (12) and noting that  $w_k^{1-\nu}(n) \cong w_k^{1-\nu}(n-1)$ , we obtain the final update relation for the weight vectors of the FLMS algorithm as:

$$w_k(n+1) = \begin{cases} w_k(n) + \mu_1 e(n)u(n-k) + \mu_f e(n)u(n-k) \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} & w_k \geq 0 \\ w_k(n) + \mu_1 e(n)u(n-k) - \mu_f e(n)u(n-k) \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} & w_k < 0 \end{cases} \quad (18)$$

where  $\Gamma(\cdot)$  denotes the gamma function. It is also noteworthy that from the standpoint of implementation, we have used here a modified version of the update rule as compared with that given in [10]. The above equation can be written as follows:

$$w_k(n+1) = w_k(n) + \left( \mu_1 + \mu_f \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} \text{sign}(w_k(n)) \right) (e(n)u(n-k)), \quad (19)$$

where  $\text{sign}(\cdot)$  is the sign function. Here, it is clearly observed that the update rule of FLMS uses a modified step-size as compared with the conventional LMS algorithm given in Eq. (7).

#### E. Normalized Fractional LMS

The NFLMS idea is based on the fact that the normalized version of LMS algorithm has better performance than the standard LMS method. Furthermore, it has been shown that the fractional LMS (FLMS) algorithm, which is an improved version of the conventional LMS, has faster convergence rate than LMS [10]. Thus, it is expected that using normalized version of FLMS (i.e., NFLMS) instead of FLMS leads to a better performance of adaptive filter. The update rule for NFLMS [12] is given by:

$$w_k(n+1) = w_k(n) + \left( \mu_1 + \mu_f \frac{w_k^{1-k}(n)}{\Gamma(2-\nu)} (\text{sign}(w_k(n))) \right) \left( \frac{e(n)\mathbf{u}(n)}{\delta + \|\mathbf{u}(n)\|^2} \right). \quad (20)$$



Here,  $\nu$  is the fractional order,  $\mu_1$  is the first order step size,  $\mu_f$  is the fractional-order step-size, and  $\delta > 0$ .

F. Affine Projection Algorithm

The affine projection algorithm (APA) [4] was derived as a generalization of the well-known normalized least-mean squares (NLMS) algorithm, in the sense that each tap weight vector update of NLMS is viewed as a  $1 \times L$  affine projection, whereas, in APA, the projections are made in multiple dimensions. Consider an adaptive filter of length  $L$ , defined by the coefficients vector  $\mathbf{w}(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]$ , where  $n$  is the discrete-time index. By substituting:

$$\hat{\mathbf{R}}_{uu}(n) = \mathbf{U}(n)\mathbf{U}^T(n), \tag{21}$$

$$\hat{\mathbf{r}}_{du}(n) = \mathbf{d}(n)\mathbf{U}(n), \tag{22}$$

in the update rule of Newton recursion (Eq. (4)), the equations that define the classical APA are obtained as follows:

$$\mathbf{e}(n) = \mathbf{d}(n) - \mathbf{U}^T(n)\mathbf{w}(n-1), \tag{23}$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mu \mathbf{U}(n) [\delta \mathbf{I}_p + \mathbf{U}^T(n)\mathbf{U}(n)]^{-1} \mathbf{e}(n), \tag{24}$$

where  $\mathbf{d}(n) = [d(n), d(n-1), \dots, d(n-P+1)]^T$  is a vector containing the most recent  $P$  samples of the reference (or desired) signal, with  $P$  denoting the projection order, the matrix

$$\mathbf{U}(n) = \begin{bmatrix} u(n) & \dots & u(n-P-1) \\ \vdots & \ddots & \vdots \\ u(n-L+1) & \dots & u(n-P-L) \end{bmatrix}$$

is the input signal matrix, the constant  $\mu$  denotes the step-size parameter,  $\delta$  is the regularization constant, and  $\mathbf{I}_p$  is the  $P \times P$  identity matrix. Here, it can be easily noticed that for  $P = 1$ , the update rule for the normalized least-mean-squares (NLMS) algorithm is obtained.

III. PROPOSED METHOD

In this paper, we propose new adaptive filtering technique based on fractional derivative of the mean-square-error cost function.

A. Fractional Affine Projection Algorithm

As discussed in previous section, affine projection algorithms are made in multiple dimensions which cause better approximations of  $\mathbf{r}_{du}$  and  $\mathbf{R}_{uu}$  in Newton recursive Eq. (4). By rewriting the Newton recursive equation, we propose the idea of fractional affine projection by adding the fractional derivative term to the conventional elements of the update equation in the affine projection algorithm.

To obtain the final update rule for the proposed adaptive filtering method, we should first compute the fractional derivative of the cost function given in Eq. (2). For this purpose, Eq. (2) is expanded as:

$$J(n) = E[d^2(n)] - 2E \left\{ d(n) \sum_{i=0}^{M-1} w_i(n)u(n-i) \right\} + E \left\{ \sum_{i=0}^{M-1} w_i(n)u(n-i) \sum_{j=0}^{M-1} w_j(n)u(n-j) \right\}. \tag{25}$$

Now, by applying the fractional derivative operator to the above relation, we obtain

$$\frac{\partial^\nu J(n)}{\partial w_k^\nu} = -2 \frac{\partial^\nu E \left\{ d(n) \sum_{i=0}^{M-1} w_i(n)u(n-i) \right\}}{\partial w_k^\nu} + \frac{\partial^\nu E \left\{ \sum_{i=0}^{M-1} w_i(n)u(n-i) \sum_{j=0}^{M-1} w_j(n)u(n-j) \right\}}{\partial w_k^\nu}. \tag{26}$$

Interchanging the expectation and the fractional derivative operators gives

$$\frac{\partial^\nu J(n)}{\partial w_k^\nu} = -2E \left\{ d(n) \frac{\partial^\nu \left( \sum_{i=0}^{M-1} w_i(n)u(n-i) \right)}{\partial w_k^\nu} \right\} + E \left\{ \frac{\partial^\nu \left( \sum_{i=0}^{M-1} w_i(n)u(n-i) \sum_{j=0}^{M-1} w_j(n)u(n-j) \right)}{\partial w_k^\nu} \right\}, \tag{27}$$

This can be simplified as:

$$\begin{aligned} \frac{\partial^\nu J(n)}{\partial w_k^\nu} &= -2E \left\{ d(n) \frac{\partial^\nu (w_i(n)u(n-i))}{\partial w_k^\nu} \right\} \\ &+ E \left\{ \frac{\partial^\nu \left( \sum_{j=0}^{M-1} \sum_{i=0}^{M-1} w_i(n)w_j(n)u(n-i)u(n-j) \right)}{\partial w_k^\nu} \right\} \\ &= -2E \left\{ d(n) \frac{\partial^\nu (w_i(n)u(n-i))}{\partial w_k^\nu} \right\} \\ &+ 2E \left\{ \sum_{\substack{j=0 \\ j \neq k}}^{M-1} \frac{\partial^\nu w_k(n)}{\partial w_k^\nu} w_j(n)u(n-k)u(n-j) \right\} \\ &+ E \left\{ \frac{\partial^\nu (w_k(n))^2}{\partial w_k^\nu} (u(n-k))^2 \right\}. \end{aligned} \tag{28}$$

By applying the fractional derivative operator of Eq. (17), the above relation can be rewritten as:



$$\begin{aligned} \frac{\partial^v J(n)}{\partial w_k^v} &= -2E \left\{ d(n) \frac{1}{\Gamma(2-\nu)} w_k^{1-\nu}(n) u(n-k) \right\} \\ &+ 2E \left\{ \frac{w_k^{1-\nu}(n) u(n-k)}{\Gamma(2-\nu)} \sum_{\substack{j=0 \\ j \neq k}}^{M-1} w_j(n) u(n-j) \right\} \\ &+ E \left\{ \frac{\Gamma(3)}{\Gamma(3-\nu)} w_k^{2-\nu}(n) (u(n-k))^2 \right\} \\ &= -2E \left\{ d(n) \frac{1}{\Gamma(2-\nu)} w_k^{1-\nu}(n) u(n-k) \right\} \\ &+ 2E \left\{ \frac{w_k^{1-\nu}(n) u(n-k)}{\Gamma(2-\nu)} \sum_{\substack{j=0 \\ j \neq k}}^{M-1} w_j(n) u(n-j) \right\} \\ &+ 2E \left\{ \frac{1}{(2-\nu)\Gamma(2-\nu)} w_k^{2-\nu}(n) (u(n-k))^2 \right\}. \end{aligned} \tag{29}$$

From the stand point of the algorithm implementation, we use here the following approximation:

$$\begin{aligned} 2E \left\{ \frac{1}{(2-\nu)\Gamma(2-\nu)} w_k^{2-\nu}(n) (u(n-k))^2 \right\} &\square \\ 2E \left\{ \frac{1}{\Gamma(2-\nu)} w_k^{2-\nu}(n) (u(n-k))^2 \right\}. \end{aligned} \tag{30}$$

Using this approximation in Eq. (28), we obtain

$$\begin{aligned} \frac{\partial^v J(n)}{\partial w_k^v} &\approx -2E \left\{ \frac{1}{\Gamma(2-\nu)} w_k^{1-\nu}(n) d(n) u(n-k) \right\} \\ &+ 2E \left\{ \frac{w_k^{1-\nu}(n)}{\Gamma(2-\nu)} \sum_{j=0}^{M-1} w_j(n) u(n-j) u(n-k) \right\} \\ &= -2 \frac{w_k^{1-\nu}(n)}{\Gamma(2-\nu)} E \{ d(n) u(n-k) \} \\ &+ 2 \frac{w_k^{1-\nu}(n)}{\Gamma(2-\nu)} \sum_{j=0}^{M-1} w_j(n) E \{ u(n-j) u(n-k) \}. \end{aligned} \tag{31}$$

Now, based on the update rule for the steepest-descent method (i.e., Eq. (3)), we derive a modified relation for the steepest method as follows:

$$w_k(n+1) = w_k(n) - \tilde{\mu}_1 \frac{\partial J(n)}{\partial w_k} - \tilde{\mu}_f \frac{\partial^v J(n)}{\partial w_k^v}, \tag{32}$$

where the last term (i.e.,  $\frac{\partial^v J(n)}{\partial w_k^v}$ ) is given by Eq. (31).

Comparing Eq. (4) with the update rule for steepest-descent method (i.e., Eq. (3)), it is clearly observed that the Newton recursive method is modified version of the steepest-descent method in the sense of different step-sizes used. In the same manner, we choose  $\tilde{\mu}_1$  and  $\tilde{\mu}_f$  in Eq. (31) as follows:

$$\tilde{\mu}_1 = (\mathbf{R}_{uu} + \varepsilon \mathbf{I})^{-1} \mu_1 \tag{33}$$

$$\tilde{\mu}_f = (\mathbf{R}_{uu} + \varepsilon \mathbf{I})^{-1} \mu_f \tag{34}$$

By substituting Eq. (30) in Eq. (31), and defining the following vectors:

$$\mathbf{Z} = (\mathbf{R}_{uu} + \varepsilon \mathbf{I})^{-1} \mathbf{r}_{du} \tag{35}$$

$$\mathbf{G} = (\mathbf{R}_{uu} + \varepsilon \mathbf{I})^{-1} \mathbf{R}_{uu} \mathbf{w}, \tag{36}$$

We obtain the modified Newton recursive update rule in its simplified form as:

$$w_k(n+1) = \begin{cases} w_k(n) + \mu_1 \{ \mathbf{Z}(k) - \mathbf{G}(k) \} \\ + \mu_f \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} \{ \mathbf{Z}(k) - \mathbf{G}(k) \} & w_k \geq 0 \\ w_k(n) + \mu_1 \{ \mathbf{Z}(k) - \mathbf{G}(k) \} \\ - \mu_f \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} \{ \mathbf{Z}(k) - \mathbf{G}(k) \}. & w_k < 0 \end{cases} \tag{37}$$

The above relation can equally be used as the update relation for the  $k^{th}$ -element of the coefficient vector  $\mathbf{w}$  in the proposed fractional affine projection algorithm (FAPA), provided that we use estimated values of  $\mathbf{r}_{du}$  and  $\mathbf{R}_{uu}$  as given in Eqs. (21) and (22). Based on this assumption, the new values of vectors  $\mathbf{Z}$  and  $\mathbf{G}$  can be given as:

$$\mathbf{Z} = \frac{1}{P} \mathbf{U}^T (\mathbf{U} \mathbf{U}^T + \varepsilon \mathbf{I}_p)^{-1} \mathbf{d}, \tag{38}$$

$$\mathbf{G} = \frac{1}{P} \mathbf{U}^T (\mathbf{U} \mathbf{U}^T + \varepsilon \mathbf{I}_p)^{-1} \mathbf{U}^T \mathbf{w}. \tag{39}$$

Equation (37) can be rewritten as follows:

$$w_k(n+1) = \begin{cases} w_k(n) + \left( \mu_1 + \mu_f \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} \right) \{ \mathbf{Z}(k) - \mathbf{G}(k) \} & w_k \geq 0 \\ w_k(n) + \left( \mu_1 - \mu_f \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} \right) \{ \mathbf{Z}(k) - \mathbf{G}(k) \}, & w_k < 0 \end{cases} \tag{40}$$

This relation can be summarized as:

$$w_k(n+1) = w_k(n) + \left( \mu_1 + \mu_f \frac{|w_k(n)|^{1-\nu}}{\Gamma(2-\nu)} (\text{sign}(w_k(n))) \right) \{ \mathbf{Z}(k) - \mathbf{G}(k) \}. \tag{41}$$

The above equation shows that the new algorithm modifies the step-size in a nonlinear way. The nonlinear step-size relationship given in parenthesis can be expressed by the following function:

$$f_{\text{step-size}}(w) = (1 + |w|^{1/2} \text{sign}(w)), \tag{42}$$

where  $\nu = 0.5$  and  $\mu_1 = \mu_f = 1$ . The step-size function of Eq. (42) is plotted in Fig. 2.





## IV. EVALUATIONS

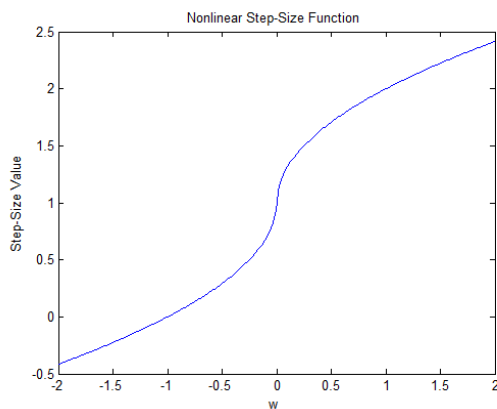
## A. Experimental Conditions and Databases

For our simulations, we use speech signals from the NOIZEUS database [15]. As noise reference, we take noises from the NOISEX-92 database [16].

In order to produce noisy speech, we use two strategies. First, we use a 30<sup>th</sup>-order FIR filter as acoustic path and produce noisy signal (i.e.,  $d(n)$ ) using a random signal as speech signal  $s(n)$ . In the second strategy, to simulate real conditions, the same filter is used with a real speech signal as  $s(n)$ . Table I shows the parameters used in the implementation of different algorithms.

## B. Simulation Results

In order to assess our proposed method, we compare the simulation results of algorithms based on different subjective and objective criteria. In the first group of experiments, we compare performances of different algorithms by plotting their learning curves (i.e., MSE plots). For this case, we use random signal with normal distribution as clean input signal  $s(n)$ , white noise as noise signal  $u(n)$ , a 30<sup>th</sup>-order type I FIR



**Figure 2:** The nonlinear step-size function,  $f_{\text{step-size}}$ , plotted as a function of  $w$ .

**TABLE I.** Parameters used for the implementation of algorithms

Algorithms	Parameters	Range of Values
LMS, NLMS, APA	step size ( $\mu_1$ )	0.05
FLMS, NFLMS, FAPA	step size ( $\mu_1$ )	0.05
	fractional step-size ( $\mu_f$ )	0.05
	fractional derivation order ( $\nu$ )	0.5
NLMS, NFLMS	$\delta$	0.001
APA, FAPA	$P$	20

filter as acoustic path, and a 30<sup>th</sup>-order adaptive filter. Fig. 3 shows the corresponding plots for the LMS, FLMS, NLMS, and NFLMS algorithms, obtained by averaging the results over 500 runs. As the result of this simulation shows, the NFLMS algorithm

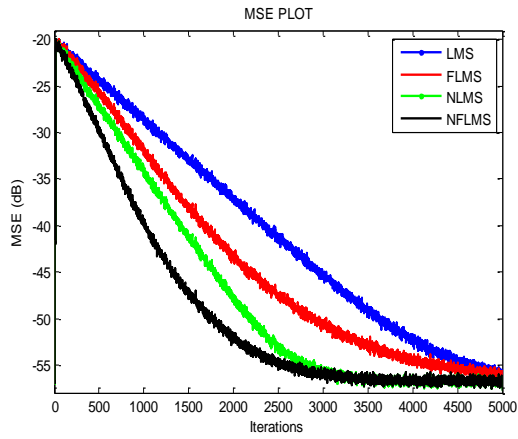
converges faster than other algorithms. Fig. 4 shows the corresponding plots for the APA and FAPA algorithms, obtained by averaging the results over 500 runs. As it is shown, our proposed method (i.e., FAPA) converges faster than the other algorithm. Finally, Fig. 5 shows the MSE plots for the LMS, FLMS, NLMS, NFLMS, APA and FAPA algorithms, obtained again by averaging the results over 500 runs. Here, it is clearly seen that the proposed method (i.e., FAPA) converges faster than other algorithms.

In the second group of simulations, we investigate the performance of the proposed method for the case of real speech signals. For this purpose, we consider the LMS, FLMS, NLMS, NFLMS, APA, and FAPA methods as adaptive filters. The evaluations of the methods are performed by inspecting the quality of enhanced speech signal both in objective and subjective manner. In this part of simulations, we use room impulse response to simulate real conditions. As noise signal, babble noise with SNRs of -10 dB, -5 dB, 0 dB, 5 dB, and 10 dB and white noise with SNRs of -10 dB, -5 dB, 0 dB, 5 dB, and 10 dB are used.

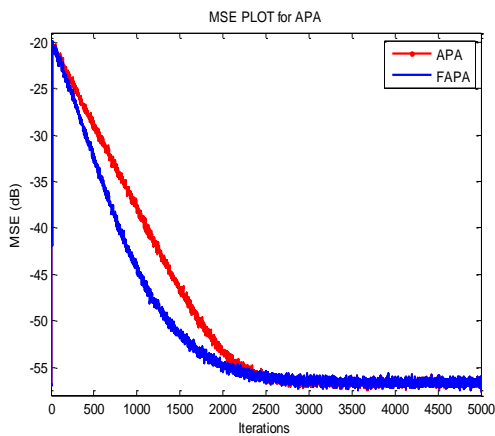
As objective evaluation criteria, we use the segmental SNR and PESQ tests [17], [18]. The results are shown in Figures 6, 7, 8, and 9 for different noise sources and different input SNR values. The results are averaged among 10 speech signal mixtures. As it can be seen from the figures, speech enhanced by FAPA has the best quality, compared with other methods. This is in accordance with the MSE evaluation results obtained by using random clean signal.

In order to assess the algorithms by subjective tests, we use the Multi Stimulus test with Hidden Reference and Anchor (MUSHRA), which is an ITU-R Recommendation BS.1534-1 [19] as implemented in [20], [21]. The subjects (i.e., human listeners) are provided with test utterances plus one reference and one hidden anchor, and are asked to rate the different signals on a scale of 0 to 100, where 100 is the best score. The listeners are permitted to listen to each sentence several times and always have access to the clean signal reference. The test signals are the same as those, used for the objective evaluation. Two types of noises (i.e., white noise and babble noise) are used in our listening tests. A total of 14 listeners (4 females and 10 males between ages of 18 to 60) have participated in these tests. Figures 10 and 11 show the results of subjective listening tests for each algorithm and different noise types.

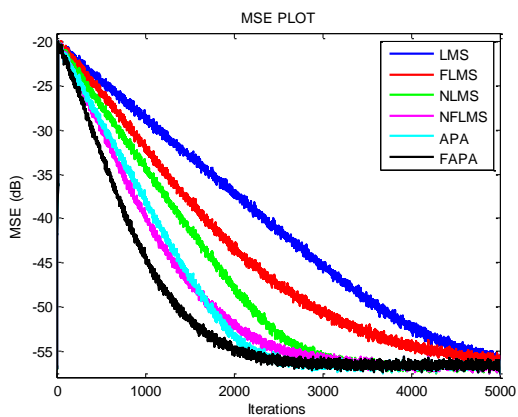
By examining the results of listening tests, we observe that the FAPA method produces the highest speech quality in speech enhancement system, as compared with other simulated algorithms. The superior performance of the FAPA method is in agreement with the results obtained during the objective evaluation tests, and is again in accordance with the MSE learning curves obtained by random clean signal.



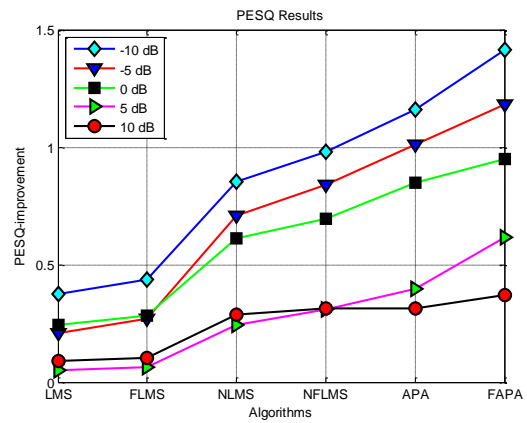
**Fig. 3:** MSE plots of LMS, FLMS, NLMS, and NFLMS with random input noise and 30-order FIR filter as acoustic path, averaged over 500 generations.



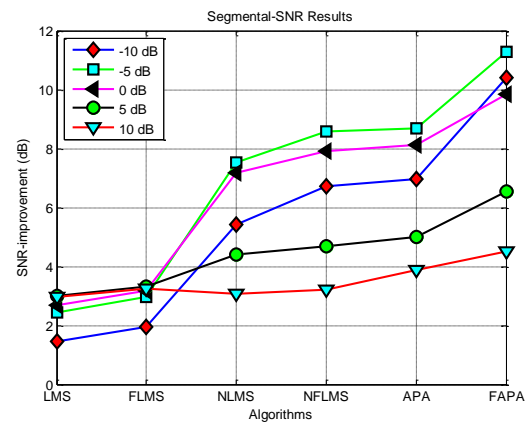
**Fig. 4:** MSE plots of APA and FAPA with random input noise and 30-order FIR filter as acoustic path, averaged over 500 generations..



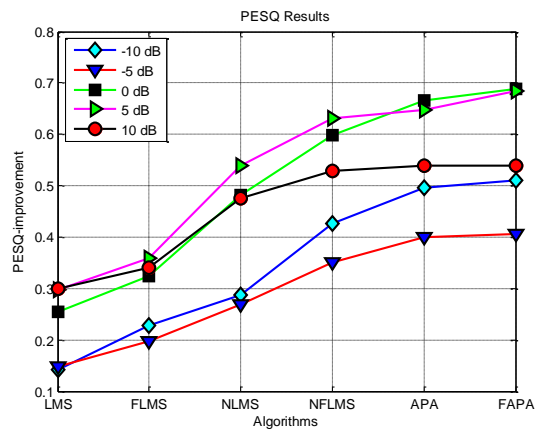
**Fig. 5:** MSE plots of LMS, FLMS, NLMS, NFLMS, APA, and FAPA with random input noise and 30-order FIR filter as acoustic path, averaged over 500 generations.



**Fig. 6:** PESQ-improvement of the enhanced speech obtained by LMS, FLMS, NLMS, NFLMS, APA, and FAPA using babble input noise and 30-order FIR filter as acoustic path under different SNR conditions.

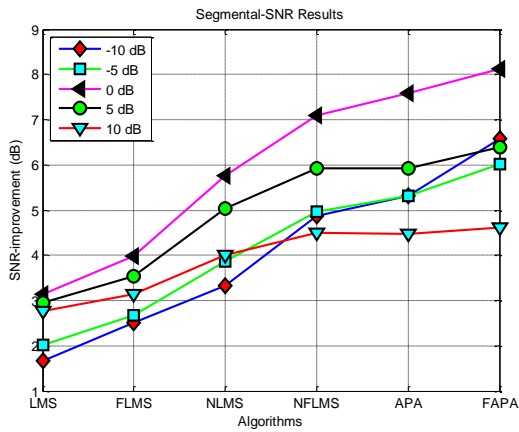


**Fig. 7:** SNR-improvement of the enhanced speech obtained by LMS, FLMS, NLMS, NFLMS, APA, and FAPA using babble input noise and 30-order FIR filter as acoustic path under different SNR conditions.

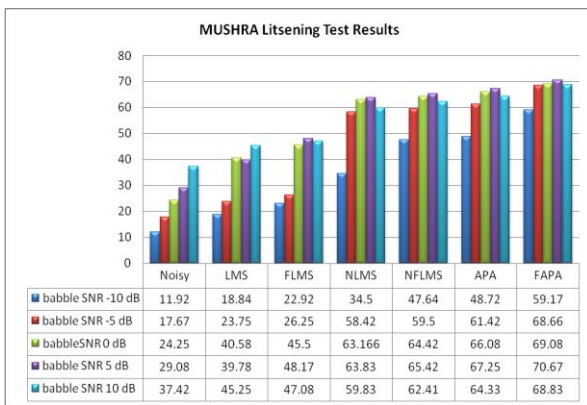


**Fig. 8:** PESQ-improvement of the enhanced speech obtained by LMS, FLMS, NLMS, NFLMS, APA, and FAPA using white input noise and 30-order FIR filter as acoustic path under different SNR conditions.

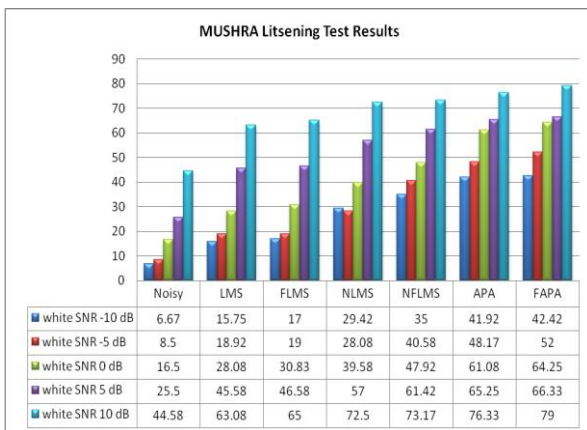




**Fig. 9:** SNR-improvement of the enhanced speech obtained by LMS, FLMS, NLMS, NFLMS, APA, and FAPA using white input noise and 30-order FIR filter as acoustic path under different SNR conditions.



**Fig. 10:** Averaged MOS results of the enhanced speech obtained by LMS, FLMS, NLMS, NFLMS, APA, and FAPA using babble input noise and 30-order FIR filter as acoustic path under different SNR conditions.



**Fig. 11:** Averaged MOS results of the enhanced speech obtained by LMS, FLMS, NLMS, NFLMS, APA, and FAPA using white input noise and 30-order FIR filter as acoustic path under different SNR conditions.

V. CONCLUSIONS

In this paper, we propose new adaptive filtering method by combining the idea of fractional derivative term in updating equations and conventional affine projection algorithm. The new algorithm has been employed in the area of dual-channel speech enhancement.

To evaluate the performance of this new idea, we first examine the simulations of learning curves (i.e., MSE plots) using random signal instead of clean speech signal. As it can be inferred by the behaviors of the MSE plots, it can be verified that the idea of fractional derivative leads to improved performance in the sense of convergence rate in the adaptive speech enhancement. The comparison of MSE results shows clearly that the FAPA algorithm has the best performance among all the simulated algorithms.

In the second stage, we use a real speech signal in our simulations and investigate the quality of the enhanced speech, both objectively and subjectively. To this aim, we compare FAPA, as selected by the MSE evaluations, with other methods. As objective evaluation, SNR and PESQ improvements obtained by different methods are compared. From objective results, we conclude that the speech enhanced by FAPA has the highest quality.

To assess the performance of FAPA subjectively, listening tests are conducted for the same enhanced speech signals as used in the objective evaluation. The results show once again that the speech signal enhanced by FAPA gives the highest quality among the all simulated methods.

In general, the powerful aspect of our proposed method appears to be its high convergence rate which has not been proved mathematically in this work. This step can be considered as our future work.

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