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A Fairness-Guaranteed Game-Theoretic Perspective in Multi-User Interference Channel

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Abstract—**In this paper, a novel game theoretic perspective with pricing scheme over a multi-user Gaussian interference channel is presented. The Kalai-Smorodinsky bargaining solution (KSBS) as a measure for guaranteeing fairness in resource allocation among users on the weak Gaussian interference channel is investigated. By using the treating interference as noise (TIN) scenario and applying proper prices for the transmit power of each user the result of the proposed game settles on a unique fair point. Also, an iterative algorithm is proposed that converges to the KSBS when users update their transmit powers and prices. Numerical results confirm analytical development.**

Keywords- Gaussian Interference Channel; Game Theory; Pricing Scheme; Nash Equilibrium; Kalai-Smorodinsky Bargaining Solution; Fairness.

I. INTRODUCTION

The role of the interference channel is significant in modeling the impact of interference in nowadays wireless communication networks like cellular networks, ad-hoc networks, sensor networks and so on [1]. Wireless mediums have become interference limited rather than noise limited as the density of the users is increasing [2].

Despite lots of studies [3]-[8] on the interference channel where multiple uncoordinated links share a common communication medium, the capacity region characterization of the interference channel is not known properly.

The strong interference regime is considered in [3] and [4]. In [5], three regimes of Gaussian interference channel (GIC) as weak, one-sided and mixed are studied and the sum capacity for a certain range of

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channel parameters is derived. In [6], for a two-user Gaussian interference channel, an outer bound is derived by considering that the side information is given to each receiver to decode both transmitters` messages and also a bound for the weak interference regime is obtained. The best known achievable rate region is due to Han-Kobayashi scheme [7] by combination the ideas of time sharing and rate splitting, where users are allowed to split their transmit power into two parts: the private part and the common part. In [8] by making significant progress towards the general capacity region, a new outer bound on the capacity region is obtained and it has been shown that for the Gaussian inputs the Han-Kobayashi scheme without time sharing comes within one bit of the derived outer bound.

One of the low complexity methods to deal with the interference channel is to allow the communication links to treat each other's interference as an addition to the noise floor. In [9], it has been shown that treating interference as noise (TIN) not only is optimal for the entire generalized degree of freedom region but also it reaches within constant gap of the entire capacity region.

On the other hand, game theoretic approaches have been received significant attention for utility maximization and resource allocation in the wireless communication networks [10]. Most game-theoretic approaches in wireless communication networks are grouped into two categories: Non-cooperative games [11]-[13] and Cooperative games [14]-[15].

In the recent decade, game-theoretic approaches also have been used in the interference channel and interference management systems [16]-[22].

On the interference channel, the operating point is chosen to achieve efficiency and fairness in resource allocation [16]. The behavior of the users in the interference channel is generally rational and selfish, which means that they have incentive to obtain more achievable rates and they only try to maximize their own utility without considering the whole system utility. In these environments where users are selfish, there is no guarantee that the efficient and fair operating point is achieved [16]. Sometimes the efficient and fair operating point is obtained by users` cooperation in choosing power transmission, codebook and rate allocation. But in practice users may not have motivation to cooperate with each other. Some techniques have been used to control the selfish and non-cooperative behavior of the users [16]. Also it has been shown in [16] that unfair situations on the spectrum sharing for multiple interfering systems may occur because of the asymmetry and selfishness behavior of the users. By proposing self-enforcing spectrum sharing rules efficient and fair situations have been obtained.

In [17], the notion of Nash equilibrium region has been exactly characterized in a non-cooperative one-shot game on the two-user linear deterministic interference channel where the utility of each user has been defined as its achievable rate. Nash bargaining solution (NBS) is used in [18] as a tool to get fair information rates and to obtain a specific point on the rate region of the interference channel which is better than other points in the context of bargaining theory.

Also, pricing schemes have drawn more attention in resource allocation and power control. For mentioning a few, in [19] a distributed pricing scheme has been used for the MIMO interference channel that reflects compensation paid by the other users for their interferences.

The paper [20] has studied a bargaining approach on the 2-user Gaussian interference channel, which is also the motivation of this paper. Authors in [20] have proposed a two-phase mechanism for the selfish users to incentive them to coordinate their transmission strategies. The phase 1 includes choosing a simple Han-Kobayashi type scheme with Gaussian codebooks and fixed power split and the phase 2 includes bargaining over the achievable rate region to obtain a fair operating point.

In [21], in the 2-user weak GIC, a utility for users has been defined by using pricing scheme to control non-cooperative behavior of them. They have defined the utility of each user as its achievable rate minus the cost of each unit transmission power. There, the fair operating point has been defined as proportional fair point.

In this paper, which is an extended version of the paper [22], we consider the N-user weak GIC and use the power pricing scheme in each user's utility to force them to agree in operating at a fair point. We use the Kalai-Smorodinsky bargaining solution (KSBS) [23]-[24] instead of proportional fair solution and NBS [25]-[26], because in considering the fairness point, KSBS is fairer than proportional fair solution and NBS [23]-[24]. Therefore, in what follows two games are considered. In the first game, we show that users` utility functions are their achievable rates using TIN. The users want to maximize their utilities by choosing power transmission strategy on multi-user GIC non-cooperatively and selfishly. Thus, the KSBS fair operating point is not achieved. Therefore, for punishing users and forcing them to choose their power transmission strategies to obtain the KSBS, we need to change the utility functions of the users. A price-based utility function for each user is defined. For all ranges of given prices (the price of each unit of transmit power) we define the best response function for the transmission power. Then we substitute the obtained best response function in the utility function. For the second game, we put the obtained utility function in the KSB problem and by solving the optimization problem for the given prices, the best transmit power for achieving KSBS is obtained theoretically. At last, the best prices of all users participating in the bargaining in order to reach the KSBS are obtained numerically through a proposed iterative algorithm.

There are some significant differences and improvement in comparing our approach in this paper and the paper [20]. One of the differences is the bargaining operating point where the NBS has been used as system operating point in [20], while in this paper, we use KSBS for introducing our efficient and fair operating point which is fairer than NBS [23]-[24]. Also this study is among N users in the GIC while the game in [20] is among 2 users. Another difference is in

the users' incentivizing techniques. As it is mentioned in [20], in information theoretic approaches, full cooperation is assumed among users for the rate selection. In the environments where there is no cooperation among the users, the NBS may not necessarily be the agreement reached in practice because a centralized management is required to ensure that all users agree to operate at the fair point that is the solution point of Nash bargaining scheme. In many communication channels, having such a manager is lacking. Thus, in [20], a non-cooperative bargaining approach as alternating-offer bargaining game (AOBG) has been proposed to take into account the cost of delay of each user in bargaining and control the users' actions to reach the fair operating point. As mentioned above, we use price-based punishment techniques to force the users to choose their power transmission strategy in such a way that efficient and fair operating point is achieved. Also in the game formulation in [20], an extensive form game with perfect information has been investigated, while here a strategic form game is used.

The significant difference between [21] and this paper`s scenario, in addition to the extension in the number of the users and the definition of fair operating point, is that in [21] the price of each unit of the transmission power has been obtained by a price function which the network manager derives it and announces the transmission power prices to the users. In the scenario of this paper, there is no manager and users obtain their transmission power prices in a distributed manner with an iterative algorithm which is more practical than centralized scenarios for implementing in the communication networks.

The rest of the paper is organized as follows. In Section II, the system model and some preliminaries are described. In Section III, a non-cooperative power control game without pricing is considered and then by applying pricing schemes, the result of the new game is presented. In Section IV, KSBS on the N-user GIC is obtained and an iterative algorithm is proposed. In Section V numerical results are presented that confirm the analytical development. Finally, in Section VI, the paper is concluded.

Notation: All logarithms are to the base 2.

II. System Model and Preliminaries

A. Channel Model

The multi-user Gaussian interference channel with N transmitters and their corresponding receivers is depicted in Fig.1 and is formulated by the following,

$$Y_{j,t} = \sum_{i=1}^{N} h_{ji} X_{i,t} + Z_{j,t} \qquad j \in \{1, \dots, N\} \quad (1)$$

where $X_{i,t}$ and $Y_{j,t}$, $t = 1, \dots, n$ represent the input and output at transmitter i and receiver j $(i, j \in \{1, \dots, N\})$ at time t, respectively, and $Z_{j,t}$ is assumed to be the independent additive complex white Gaussian noise with zero mean and variance of σ_n^2. h_{ji} is the channel gain between receiver j and transmitter i.

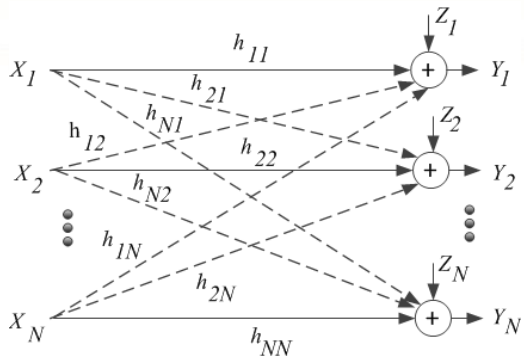

Fig. 1. Multi-User Gaussian interference channel.

Each transmitter i, transmits its message W_i to the related receiver. Receiver i, $(i = 1, \dots, N)$ is only interested in the message sent by transmitter i.

For a given block length n, user i sends a message $W_i \in \{1,2, \dots, 2^{nR_i}\}$ by encoding it to a codeword $X_i^{(n)} = (X_{i,1}, X_{i,2}, \dots, X_{i,n})$.

The codewords are real-valued and satisfy the block average power constraints given by,

$$\frac{1}{n}\sum_{t=1}^{n} |X_{i,t}|^2 \le P_i^{max} \ , i = 1, \dots, N. \qquad (2)$$

Receiver i observes the channel output $Y_i^{(n)} = (Y_{i,1}, \dots, Y_{i,n})$ and uses a decoding function $f_i : \mathbb{R}^n \longrightarrow \{1,2, \dots, 2^{nR_i}\}$ to get the estimate $\widehat{W}_i$ of the transmitted message W_i. The probability of error at each receiver is defined by the expression,

$$p_{e,i}^n = \mathrm{P}\left\{f_i\left(Y_i^{(n)}\right) \neq W_i\right\} \quad i = 1, \dots, N. \qquad (3)$$

and $p_e^n = \max\{p_{e,1}^n, p_{e,2}^n, \dots, p_{e,N}^n\}$.

A rate tuple $(R_1, \dots, R_N)$ is said to be achievable if there is a sequence of $(2^{nR_1}, \dots, 2^{nR_N}, n)$ codes with $p_e^n \longrightarrow 0$ as $n \to \infty$. The capacity region of the interference channel is the closure of the set of all achievable rate tuples.

By TIN, the achievable rate for each transmitter-receiver pair is:

$$R_i(P_i; \mathbf{P}_{-\mathrm{i}}) = \log\left(1 + \frac{P_i |h_{ii}|^2}{\sigma_n^2 + \sum_{j \neq i} P_j \left|h_{ij}\right|^2}\right), \quad (4)$$

where $i, j \in \{1, \dots, N\}, i \neq j$.

Also in this network, it is assumed that the total available transmission power for users is limited. Thus, another restriction is defined here as

$$\sum_{i=1}^{N} P_i^{max} = P_{Total}. \qquad (5)$$

which means that the sum of maximum available power for all users is P_{Total}.

B. Review of Nash Equilibrium in a Non-Cooperative Game

A game $G = \langle \mathcal{N}, (P_i), (U_i) \rangle$ has three elements: a set of users (transmitters and their corresponding receivers) $\mathcal{N} = \{1, \dots, N\}$ as players, the strategy space

$\mathcal{P}_i = [0, P_i^{max}]$ for each user i which is the interval that contains the transmit power choices, and a utility function U_i for each strategy profile $\mathbf{P} = [P_1, \dots, P_N]^T$.

In a non-cooperative power control game $G = \langle \mathcal{N}, (P_i), (U_i) \rangle$, each user tends to maximize its utility by choosing appropriate power, so users compete for achieving more utility. Formally a non-cooperative power control game can be expressed by [11], [21] and [27],

$$\max_{P_i \in \mathcal{P}_i} U_i(P_i, \mathbf{P}_{-i}), \text{ for all } i \in \{1, \dots, N\} \quad (6)$$

where $\mathbf{P}_{-i}$ denotes the vector consisting of elements of $\mathbf{P}$ other than the ith element.

Definition 1: As in [11], [21] and [27], a transmit power profile $\mathbf{P}^* = [P_1{}^*, P_2{}^*, \dots, P_N{}^*]^T$ is the Nash equilibrium point of the non-cooperative power control game $G = \langle \mathcal{N}, (P_i), (U_i) \rangle$, if for all $i \in \{1, \dots, N\}$ and for all $P_i \in \mathcal{P}_i$, we have, $U_i(P_i{}^*, \mathbf{P}_{-i}^*) \geq U_i(P_i, \mathbf{P}_{-i}^*)$.

The most favorable strategy which is being chosen by each rational self-optimizing user is the best response to the rivals power profile $\mathbf{P}_{-i}$. So the best response can be defined as $\mathcal{B}_i(\mathbf{P}_{-i}) = \arg\max_{P_i \in \mathcal{P}_i} U_i(P_i, \mathbf{P}_{-i})$, and Nash equilibrium (NE) is a fixed point of all best responses and in other words NE is an operating point that none of the players can improve its utility by unilaterally changing its strategy.

C. Kalai-Smorodinsky Bargaining Solution (KSBS)

A famous solution to the bargaining game which could be utilized to guarantee fairness is KSBS. In KSBS, the goal is maximizing r where $r = \frac{U_i - U_i^0}{U_i^* - U_i^0}$, and U_i^* is the maximum possible utility for user i and U_i^0 is the disagreement point. The NBS is another fairness criterion [23]-[24]. However, KSBS emphasizes on the equal ideal point more than NBS and it is more reliable than NBS from the fairness criterion point of view [24]. NBS fairness is a generalized proportional fairness and this form can ensure fairness of resource allocation [26]. Also, cooperative game theories prove that there exists the unique and efficient NBS under the six axioms. The Nash bargaining problem is formulated as

$$\max_{P_i} \prod_{i=1}^{N} (U_i - U_i^0) \quad (7)$$

where U_i is the user i utility and U_i^0 is the disagreement point.

III. Game Formulation

A. Power Control Game Without Pricing

By assuming $U_i = R_i$ defined in (4), all users choose their maximum power for maximizing their utilities, so the power profile at the NE point with the assumed utility is $\mathbf{P}^* = [P_1^{max}, P_2^{max}, \dots, P_N^{max}]$.

As it has been mentioned in the weak GIC, users use TIN scenario to obtain their achievable rates. It is trivial that when users do not coordinate, each user uses TIN scenario [20]. Therefore, KSBS operating point is not necessarily obtained, because all of the users prefer using their maximum power in an uncoordinated case.

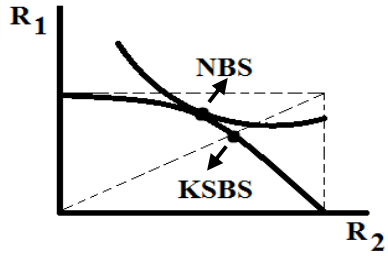

Fig. 2. KSBS vs. NBS on the two-user GIC rate region [28].

We cannot force users to adjust their powers in a way that the KSBS operating point would be established in the network.

In Fig. 2, the bargaining points mentioned above are illustrated on the rate region of the two-user GIC in weak interference scenario, which is comparing KSBS and NBS and also has been illustrated in [28].

Note that also in [20], the NBS point on the rate region of the two-user Gaussian interference channel is depicted. There, this point is achieved by using optimal Han-Kobayashi power splitting.

It is clear that the power profile $\mathbf{P}^* = [P_1^{max}, P_2^{max}]$ for two-user and $\mathbf{P}^* = [P_1^{max}, P_2^{max}, \dots, P_N^{max}]$ for N-user (which users choose the maximum power to maximize their achievable rate) do not necessarily result in NBS or KSBS. Operating on these points in practice is not guaranteed. So we add a cost function to the achievable rate of each user to force them operate on bargaining points which are the fair operating points. As mentioned above KSBS emphasizes on fairness more than NBS so we concentrate on KSBS in this paper.

By changing users` pay-off function in the next section and applying prices for transmitting messages, the power profile for achieving the maximum pay-off would be changed.

B. The Proposed Price-Based Power Control Game

As discussed in the previous section, from game-theoretic point of view, rational users without any cost, use their maximum power, so the rate tuple with this power profile is not necessarily a fair operating point and by assuming users' rate as their utilities, the results in the previous section are obtained which are not fair necessarily. When users should pay price for using power to transmit, then their selfishness behavior can be controllable. By defining a pay-off function like (8) in which the utility of each user is its achieved rate minus the price of the used power, we can investigate the behavior of users for choosing the transmit power.

$$U_i(P_i, \mathbf{P}_{-i}) = R_i(P_i, \mathbf{P}_{-i}) - \beta_i P_i \quad (8)$$

In (8), U_i, R_i and $\beta_i (i = 1, \dots, N)$ are respectively each user`s utility, rate and the price of per unit of the used power.

It is shown that for the game $G = \langle \mathcal{N}, (P_i), (U_i) \rangle$, where $U_i(P_i; \mathbf{P}_{-i}) = R_i(P_i; \mathbf{P}_{-i}) - \beta_i P_i$, a unique and Pareto-efficient NE exists for all $\beta_i \geq 0$. Also it is shown that there is a unique tuple $\beta_1, \dots, \beta_N$ that results in fairness and Pareto-efficiency at the same time.

$$b_i(\boldsymbol{P}_{-i}) = \begin{cases} P_i^{max}, & if\ 0 \le \beta_i \le R_i'(P_i^{max}) \\ \min\{P_i^{max}, \frac{1}{\beta_i ln(2)} - \frac{\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}{|h_{ii}|^2}\}, & if\ R_i'(P_i^{max}) < \beta_i \le R_i'(0) \\ 0, & if\ R_i'(0) < \beta_i \end{cases} \quad (9)$$

Lemma1 : In the game with pricing, where utility of each user defined by (8), the best response of user i to a given interference is (9), where $i, j \in \{1, \dots, N\}, i \neq j$.

Proof: For obtaining $b_i(\mathbf{P}_{-i})$, we should solve $\max_{P_i} U_i(P_i; \mathbf{P}_{-i})$. So the first and the second derivative of the proposed utility which is $U_i(P_i; \mathbf{P}_{-i}) = R_i(P_i; \mathbf{P}_{-i}) - \beta_i P_i$ with respect to P_i are used.

$$\frac{\partial U_i}{\partial P_i} = \frac{|h_{ii}|^2}{(\ln 2)\left(\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2 + P_i |h_{ii}|^2\right)} - \beta_i \quad (10)$$

$$\frac{\partial^2 U_i}{\partial {P_i}^2} = \frac{-|h_{ii}|^4}{(\ln 2)\left(\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2 + P_i |h_{ii}|^2\right)^2} \quad (11)$$

Note that $\frac{\partial^2 U_i}{\partial {P_i}^2}$ is always negative. We know that $R_i'(P_i)$, i.e., $\frac{\partial R_i}{\partial P_i}$ that is $\frac{|h_{ii}|^2}{(\ln 2)\left(\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2 + P_i |h_{ii}|^2\right)}$ is a strictly decreasing function of P_i and $R_i'(P_i^{max}) < R_i'(P_i) \le R_i'(0)$. Thus, for $0 \le \beta_i \le R_i'(P_i^{max})$, we have $\frac{\partial U_i}{\partial P_i} > 0$, so U_i is an increasing function of P_i. In this case, similar to the mentioned game in section III, the best response for each user is to transmit at its maximum power, i.e., for $0 \le \beta_i \le R_i'(P_i^{max})$ $b_i(\mathbf{P}_{-i}) = P_i^{max}$ that $i \in \{1, \dots, N\}$. For $R_i'(P_i^{max}) < \beta_i \le R_i'(0)$, the equation $\frac{\partial U_i}{\partial P_i} = 0$, or equivalently $R_i'(P_i) = \beta_i$, has the unique solution $P_i = \frac{1}{\beta_i \ln(2)} - \frac{\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}{|h_{ii}|^2}$ for $i, j \in \{1, \dots, N\}$ and $i \neq j$. As $R_i''(P_i) < 0$ for all P_i, and hence $\frac{\partial^2 U_i}{\partial {P_i}^2} < 0$, the roots of (8) maximize U_i for a given interference $\sum_{j\neq i} P_j |h_{ij}|^2$.

In this case, it is obvious that transmitter i cannot transmit more than P_i^{max} for a fixed interference so if the obtained P_i be more than P_i^{max}, the best response to $\mathbf{P}_{-i}$ is the maximum value of transmit power. Therefore, in this case we have $b_i(\mathbf{P}_{-i}) = \min\{P_i^{max}, \frac{1}{\beta_i \ln(2)} - \frac{\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}{|h_{ii}|^2}\}$.

For $R_i'(0) < \beta_i$, we have $\frac{\partial U_i}{\partial P_i} < 0$, thus U_i is a decreasing function of P_i, and the best response for transmitter i is no transmission, i.e., for $R_i'(0) < \beta_i$, $b_i(\mathbf{P}_{-i}) = 0$ for $i \in \{1, \dots, N\}$. ■

By substituting (9) in (8), we can obtain U_i^* in (12), which is the maximum utility for user i.

IV. KSBS ON GIC

A. Power Allocation Setting

In this section we are going to find the best power allocation for all users in Gaussian interference channel based on KSBS. First we assume that the prices (for a unit of power) for all users, i.e., β_i $(i = 1, \dots, N)$ are given (At the end of this section, the way of obtaining prices will be discussed) and they are not changed at each level of the game. Now, KSBS is obtained by solving,

$$\max_{P_i} r \quad (13)$$
$$\text{s.t. } r = \frac{U_i - U_i^0}{U_i^* - U_i^0}$$

where U_i is mentioned in (8). U_i^0 is the disagreement point where user i uses its maximum power, i.e., $U_i^0 = R_i(P_i^{max}, \mathbf{P}_{-i}) - \beta_i P_i^{max}$ and U_i^* is the maximum utility which is derived in (12). Similar to [25], this problem is equivalent to

$$\max_{0 \le \mathbf{P} \le \mathbf{P}^{\mathbf{max}}} \min_i \frac{U_i - U_i^0}{U_i^* - U_i^0} \quad (14)$$

where $0 \le \mathbf{P} \le \mathbf{P}^{\mathbf{max}}$ means $0 \le P_i \le P_i^{max}$ for all $i \in \{1, \dots, N\}$. The problem in (14) is also mentioned as max-equal problem, i.e., $max\{r | r = \frac{U_i - U_i^0}{U_i^* - U_i^0}\}$, subject to $0 \le P_i \le P_i^{max}$ for all $i \in \{1, \dots, N\}$.

It is obvious that if $U_i^* = U_i^0$, then $\frac{U_i - U_i^0}{U_i^* - U_i^0}$ for user i goes to infinity. Therefore, we use an assumption here

$$U_i^*(b_i(\boldsymbol{P}_{-i}), \boldsymbol{P}_{-i}) = \begin{cases} \log\left(1 + \frac{P_i^{max} |h_{ii}|^2}{\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}\right) - \beta_i P_i^{max}, & if\ 0 \le \beta_i < R_i'(P_i^{max}) \\ \log\left(1 + \frac{\frac{|h_{ii}|^2}{\beta_i ln(2)} - \sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}{\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}\right) - \frac{1}{\ln(2)} - \beta_i \frac{\sigma_n^2 + \sum_{j\neq i} P_j |h_{ij}|^2}{|h_{ii}|^2}, & if\ R_i'(P_i^{max}) \le \beta_i < R_i'(0) \\ 0, & if\ R_i'(0) \le \beta_i \end{cases} \quad (12)$$

that the users whom it is beneficial for them to use their maximum power according to their prices, (i.e., $U_i^* = U_i^0$ happens for these users), are not allowed to take part in the bargaining. So, we can find M users ($M \leq N$) that participate in the bargaining and $N - M$ users quit the game because more benefit is never acquired.

Without loss of generality, it is assumed that the M users can be sorted in the order $R_1' \geq R_2' \geq \cdots \geq R_M'$. It is investigated that with the given β_i, users whose $R_i'(P_i^{max})$ is not more than β_i would not participate in the game (i.e., the $N - M$ users mentioned above). So to force M users to take part in the game the range of β_i should be $R_{M-1}' > \beta_i \geq R_M'$. Therefore, we should find the power allocation of these M users based on KSBS with the given prices β_i for $1 \leq i \leq M$.

The other $N - M$ users use their maximum power according to their price (β_i) and do not attend in the bargaining problem. Actually, the dual of maximizing the sum rate is maximizing the minimum rate which is the fairness scheme [11].

We next obtain the solution of P_i for $1 \leq i \leq M$, according to KSBS. Therefore we have,

$$\frac{U_1 - U_1^0}{U_1^* - U_1^0} = \frac{U_2 - U_2^0}{U_2^* - U_2^0} = \cdots = \frac{U_M - U_M^0}{U_M^* - U_M^0} = r \quad (15)$$

By defining U_i^{-1} as the inverse function of $U_i(P_i, \mathbf{P}_{-i})$, we have $P_i^* = U_i^{-1}(rU_i^* + (1-r)U_i^0)$. Note that although there might be multiple solutions of P_i^*, we choose the exclusive one which holds $\sum_{i=1}^{M} U_i^{-1}(rU_i^* + (1-r)U_i^0) = P_{Total} - \sum_{k=M+1}^{N} P_k^{max}$. So it leads to the single solution r^* due to monotonicity, and further yields P_i^*.

B. *Price Setting with Distributed Iterative Resource Allocation Algorithm*

In this part, we introduce an iterative algorithm for setting the price of transmit power among users to reach the KSBS point. In this algorithm P_i^* (which is mentioned in the above subsection) and β_i^* are obtained from solving KSB. Moreover, a power allocation is Pareto-efficient, if it is not possible to increase the utility of any user without decreasing the utility of the other user, based on Pareto-efficiency definition in [11], [21] and [27]. Here, at the end of the game, there exists no power vector like $\mathbf{P}'$, such that $U_i(\mathbf{P}') \geq U_i(\boldsymbol{P}^*)$ for all users $i \in \{1, .., M\}$. So, this achieved operating point is also Pareto-efficient.

Distributed Resource Allocation Algorithm

1: Each user i, chooses initial power from $[0, P_i^{max}]$ for transmitting its message and also an initial price $\beta_i \geq 0$ is set for user i.

2: In each iteration n, power is updated according to (9).

3: The price of each unit of transmit power is updated in each iteration n according to:

$$\beta_i(n+1) = [\beta_i(n) + \kappa(P_i(n) - P_i^*)]^+.$$

where κ is the step size.

4: The utility of each user is updated from (12).

5: Jump to step 2 until $\frac{U_i(n) - U_i^0(n)}{U_i^*(n) - U_i^0(n)} - \frac{U_i(n-1) - U_i^0(n-1)}{U_i^*(n-1) - U_i^0(n-1)} > \varepsilon$ for all users $i \in \{1, .., M\}$, where ε is a small constant.

V. Numerical Results

Numerical results for the described scenario are depicted in Fig. 3, Fig. 4, Fig. 5 and Fig. 6. It is assumed that 6 users take part in the bargaining game. Coefficient κ (convergence step size) in the iterative algorithm is set to 0.01. The channel gains have been chosen randomly and all channel gains are positive and less than 1 (This is because of weak interference channel definition [5]). It is assumed that all channel gains are known to users and are fixed during the game. In Fig. 3, for each iteration the transmit power of users according to step 2 in the iterative algorithm is depicted. The maximum power for all users is equal to 3.5 watts. In Fig. 4, the updating prices of transmit power unit are illustrated. In Fig. 5 rates of users and in Fig. 6 the utility of users in the bargaining scenario during the iterative algorithm are illustrated. One can see that utilities are become equal and fairness among users are established in about 60 iterations by using KSBS to find optimal transmit powers.

VI. Conclusion

In this work, first we have assumed that users in multi-user Gaussian interference channel do not have equal rates and the users with good channel gains interfere to other users, so the fairness is not satisfied. Then, a utility is introduced with pricing for competing users and also KSBS is considered as an efficient and fair operating point. It is shown that by applying the resulted prices and power allocation during the proposed iterative resource allocation algorithm for all users the fair points are obtained. The numerical results show that all users reach the KSBS fair operating point after about 50 to 60 iterations.

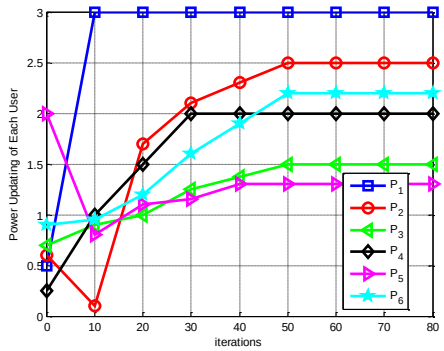

Fig. 3. The illustration of updating transmit power by users and reaching the KSBS operating point.

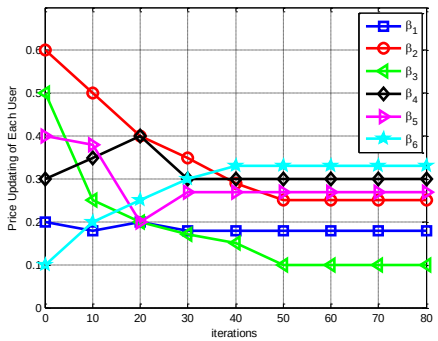

Fig. 4. The illustration of updating prices of transmit power unit.

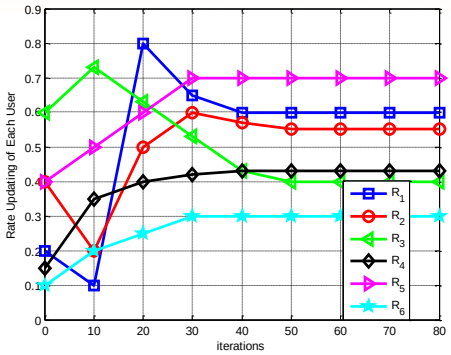

Fig. 5. The illustration of updating rates by users and reaching the KSBS operating point.

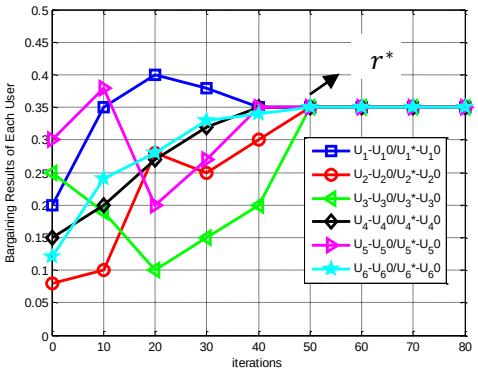

Fig. 6. The illustration of reaching the KSBS operating point.

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